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Continuous folded plate structures under uniform load.

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CONTINUOUS FOLDED PLATE STRUCTURES

UNDER UNIFORM LOAD

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ABSTRACT

In this thesis continuous folded plate structures with two spans are investigated both analytically and experimentally. The theory used to obtain the analytical solution is developed from the principles outlined by Wilhelm Flügge in his book (6). General equations are developed to find stresses and deflections of the continuous structure when it is uniformly loaded at the edges. A concrete continuous folded plate structure is analyzed experimentally and a general pattern of deflection and stress distribution is established.

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TABLE OF CONTENTS

ABSTRACT		iii
ACKNOWLEDGMENTS		iv
NOTATION		xii
Chapter		
I	INTRODUCTION	1
II	HISTORICAL DISCUSSION	9
III	METHOD OF ANALYSIS	11
IV	ANALYSIS OF PLATE bm UNDER UNIFORM LOAD S_{bm}	19
V	ANALYSIS OF PLATE am UNDER UNIFORM LOAD S_{am}	31
VI	ANALYSIS OF PLATE $_{bm}$ SUBJECTED TO UNIT MOMENT AT B	42
VII	ANALYSIS OF PLATE $_{am}$ SUBJECTED TO UNIT MOMENT APPLIED AT END B	55
VIII	PLATE m OF SPAN ABC IS MADE CONTINUOUS	65
IX	JOINT DISPLACEMENTS	69
X	ANALYTICAL SOLUTION OF EXPERIMENTAL PRISM	81
XI	EXPERIMENTAL PROGRAM	114
XII	CONCLUSION	143
LITERATURE CITED		146
VITA AUCTORIS		149

LIST OF TABLES

Table

10-1 Loads Resolved Into Components Acting in Plane of Plates 81

10-2 Longitudinal Shear Constants for Section BC . . . 85

10-3 Longitudinal Shear Constants for Section AB . . . 87

10-4 Rotation of Plates in Span BC Due to Uniform Load . 87

10-5 Rotation of Plates in Span AB Due to Uniform Load . 88

10-6 Deflections of Plates at $x = \frac{L_b}{2}$ Due to Uniform Load . 88

10-7 Deflections of Plates at $x = \frac{L_a}{2}$ Due to Uniform Load . 89

10-8 Shear Forces in lb/ft for Span BC Due to Unit Moments Applied at end B of Different Plates 90

10-9 Shear Forces in lb/ft for Span AB Due to Unit Moments Applied at End B of Different Plates 91

10-10 Rotations at B of Plates in Span BC Due to Unit Moments Applied at B 91

10-11 Rotations at B of Plates in Span AB Due to Unit Moments Applied at B 92

10-12 Deflections at Midspans Due to Unit Moments Applied at B 94

10-13 Redundant Moments at Support B 94

10-14 Deflections at Midspans Due to the Redundant Moments Applied at B 95

10-15 Deflections at Midspans of the Continuous Structure Under the Uniform Load of 100 lb/ft 95

10-16 Angles . . . . [illegible]

Table

10-17	Normal Displacements	98
10-18	Transversal Angles at $x = \frac{L_b}{2}$ and at $x = -\frac{L_a}{2}$	99
10-19	Angles at $x = \frac{L_b}{2}$ and at $x = -\frac{L_a}{2}$	99
10-20	Tangential Loads in the Planes of Four Strips Due to Unit Moment $M_r = 1$ Applied at Edge b.	100
10-21	Tangential Loads in the Planes of Four Strips Due to Unit Moment $M_r = 1$ Applied at Edge c.	100
10-22	Shear Stressed Due to Unit Moments Applied Transversely at Edges b and c	105
10-23	Rotations at $x = 0$ of Plates in Span BC Due to $M_r = 1$ at Edge b and $M_r = 1$ at Edge c	106
10-24	Deflections at $x = \frac{L_b}{2}$ of Plates in Span BC Due to $M_r = 1$ at Edge b and $M_r = 1$ at Edge c	107
10-25	Redundant Moments $M^{(r)}_{o(x=o)m}$ Due to Moments $M_r = 1$.	108
10-26	Deflections at Midspan of BC Due to Redundant Moments $M^{(r)}_{o}$ Applied to Plates at B	109
10-27	Deflections of the Plates of the Continuous Structure Due to Uniform Moments $M_r = 1$ Alternately Applied at Edge b and c	109
10-28	Normal Displacements to Each Plate Strip Due to Moments $M_r = 1$ Applied at Edges b and c Alternately . .	110
10-29	Transversal Rotation of Plate Strips at Midspans of Continuous Structure Due to $M_r = 1$	111
10-30	Relative Angle Changes at Joints Due to $M_r = 1$ Alternately Applied at Joints b and c	112
10-31	Redundant Moments Acting Uniformly Along the Edges b and c of the Continuous Structure	112

Table

10-32 Final Deflections of the Continuous Structure Due to a Uniform Load of 100 lb/ft Applied at Edges b and c . . . 113

11-1 Results of Strain Gauges 133-4

11-2 Reading of Dial Indicators 140

11-3 Reading of Dial Indicators 142

LIST OF FIGURES

Figure

1.	Continuous Folded Plate Structure	2
2.	Method of Analysis	13
3.	Uniform Load is Resolved Into the Planes of the Plates	14
4.	Plate bm Under Uniform Load S_{bm}	16
5.	Plate Cross-Section	17
6.	Longitudinal Shears on Plate bm	19
7.	Strip Element of Plate bm	20
8.	Moment Area of Cross-Section	26
9.	Volume Element of Plate bm Under Load S_{am}	27
10.	Plate $_{am}$ Under Uniform Load S_{am}	32
11.	Longitudinal Shears on Plate $_{am}$	35
12.	Strip Element of Plate $_{am}$	[illegible]
13.	Plate $_{bm}$ With Unit Moment at B	[illegible]
14.		
15.	Longitudinal Shears on Plate am with Unit Moment at B	[illegible]
16.	Volume Element of Plate $_{bm}$ with Unit Moment at B	[illegible]
17.	Plate $_{am}$ with Unit Moment at B	57
18.	Strip Element of Plate $_{am}$ with Unit Moment at B	[illegible]
19.	Normal Displacements w'_m and w''_m	70

Figure

20.	Redundant Moment M_r	72
21.	Redundant Moment M_r Resolved into Forces Acting on Planes of Plates	74
22.	Definition of the Angular Displacements w'_m, w''_m	79
23.	Data for Analytical Problem	82
24.	Data for Analytical Problem	83
25.	Loads Resolved into Forces Acting on Planes or Plates	84
26.	Longitudinal Shears	86
27.	Rotations at B Due to Unit Moment Applied at B	93
28.	Angles ϕ_m and $_m$	92
29.	Normal Displacements	98
30.	Redundant Moment M_r	101
31.	Redundant T Moment M_r Resolved into Forces Acting on Planes of Plates	102
32.	Numerical Values of S^b	103
33.	Numerical values of S^c	104
34-46.	Photos	115-127
47.	Cross-Section of Concrete Specimen	128
48.	Strain Gauge Location	129
49.	Set Up of Gauge Indicators	130
50.	Set Up of Gauge Indicators	131
51.	Modulus of Elasticity of Concrete	132

Figure

52-56.	Stress Distribution Diagrams	135-9
57.	Deflection Diagram	142
58.	Relation of Deflections Due to S_m and T_m . . .	145

NOTATION

Note: A superscript o indicates that the symbol denotes a quantity caused by uniform loads S_{am} or S_{bm}. If the letters a, b, c, d appear in the subscript they refer to the four spans shown in Fig. 2. The same letters are used in the superscript to indicate the number of the plate; in the superscript those letters replace the arabic numbers 1, 2, 3 ---- as these numbers could be easily confused with powers. If the symbol hasn't any of the letters a, b, c ----- in the subscript it refers to the continuous structure.

The first part of the analysis is carried out under the assumption that plates are hinge-connected by piano wires. In the second part of the analysis (joint displacement) the edges are made rigid by applying redundant moments $M_m^{(r)}$ where the superscript r denotes the number of the edge to which the moment is applied. These moments are resolved into $S_{m}^{(r)}$ loads (Fig. 20, Fig. 21). All resulting symbols carry the superscript r.

h_m	Height of plate m.
L_a	Length of span AB.
L_b	Length of span BC.
$M_{am}^{(o)}$	Moment of plate am (Fig. 2) due to load S_{am}.
$M_{bm}^{(o)}$	Moment of plate bm (Fig. 2) due to load S_{bm}.
$M_{am}^{(o)}$	Moment of plate am (Fig. 2) due to M_{bm} = 1 at B.

$M^{(0)}_{dm}$	Moment of plate bm (Fig. 2) due to $M_{om} = 1$ at B.
$M^{(1)}_{am}$	Moment of plate am due to shears T_{am}.
$M^{(1)}_{bm}$	Moment of plate bm due to shears T_{bm}.
$M^{(1)}_{cm}$	Moment of plate am due to shears T_{cm}.
$M^{(1)}_{dm}$	Moment of plate bm due to shears T_{dm}.
M_{om}	Redundant moment (Fig. 2) at B.
N_x, N_y, N_{xy}	Shearing stresses shown in Fig. 1. The superscript o refers to uniform loads while the superscript (1) refers to longitudinal shears T_m. For subscripts a, b, c, and d the reader may refer to Fig. 2.
$Q^{(0)}_{am}$, $Q^{(0)}_{bm}$	Shearing forces due to S_{am} and S_{bm} respectively.
$Q^{(0)}_{cm}$, $Q^{(0)}_{dm}$	Shearing forces due to unit moment at B.
S_{am}	Uniform load acting on the plane of plate am.
S_{bm}	Uniform load acting on the plane of plate bm.
t_m	Thickness of plate m.
T_m	Longitudinal shearing stress between plate m, and plate (m+1). For the superscripts the reader may refer to the note under Notation.
T^1_m or T'_m	Shearing constant related to T_m by equation (4-18).
v_{am}	Deflection of plate am under the load S_{am}.
v_{bm}	Deflection of plate bm under the load S_{bm}.
v^0_{cm}	Deflection of plate am due to unit moment at B.
v^0_{dm}	Deflection of plate bm due to unit moment at B.

$v_{cm} = M_{om} v'_{cm}$

$v_{dm} = M_{om} v'_{dm}$

γ_m Angles shown in Fig. 2.

θ_{am} Rotation of plate am due to S_{am}.

θ_{bm} Rotation of plate bm due to S_{bm}.

θ_{cm} Rotation of plate am due to unit moment at B.

θ_{dm} Rotation of plate bm due to unit moment at B.

ϕ_m Angles shown in Fig. 3.

CHAPTER I

INTRODUCTION

The prismatic folded plate structure (Fig. 1) is formed by a series of adjoining thin plane slabs mutually supporting each other and rigidly connected along their common edges. They are usually closed off at its ends by integral diaphragms. Even though there are many folded plate structures made out of plywood and some out of metal, most of them are made out of concrete because it can be easily adapted to different forms.

In reinforced concrete the unstressed material is relocated to areas of fuller utilization thus making a substantial reduction in concrete per square foot or surface and making a substantial reduction in dead load. Floor and roof systems can be shaped to accommodate ducts and utility troughs thus obtaining more economy as compared to conventional structure. Folded plates have advantages over curved shells because of the less expensive framework, easier concreting and screeding. Similar advantages exist over more complex structures such as arches and frames. Because of their stiffness, folded plates can be constructed over long spans without an increase in material requirement. For these reasons, in the last decade, folded plate construction has found increasing application for roofs of industrial buildings

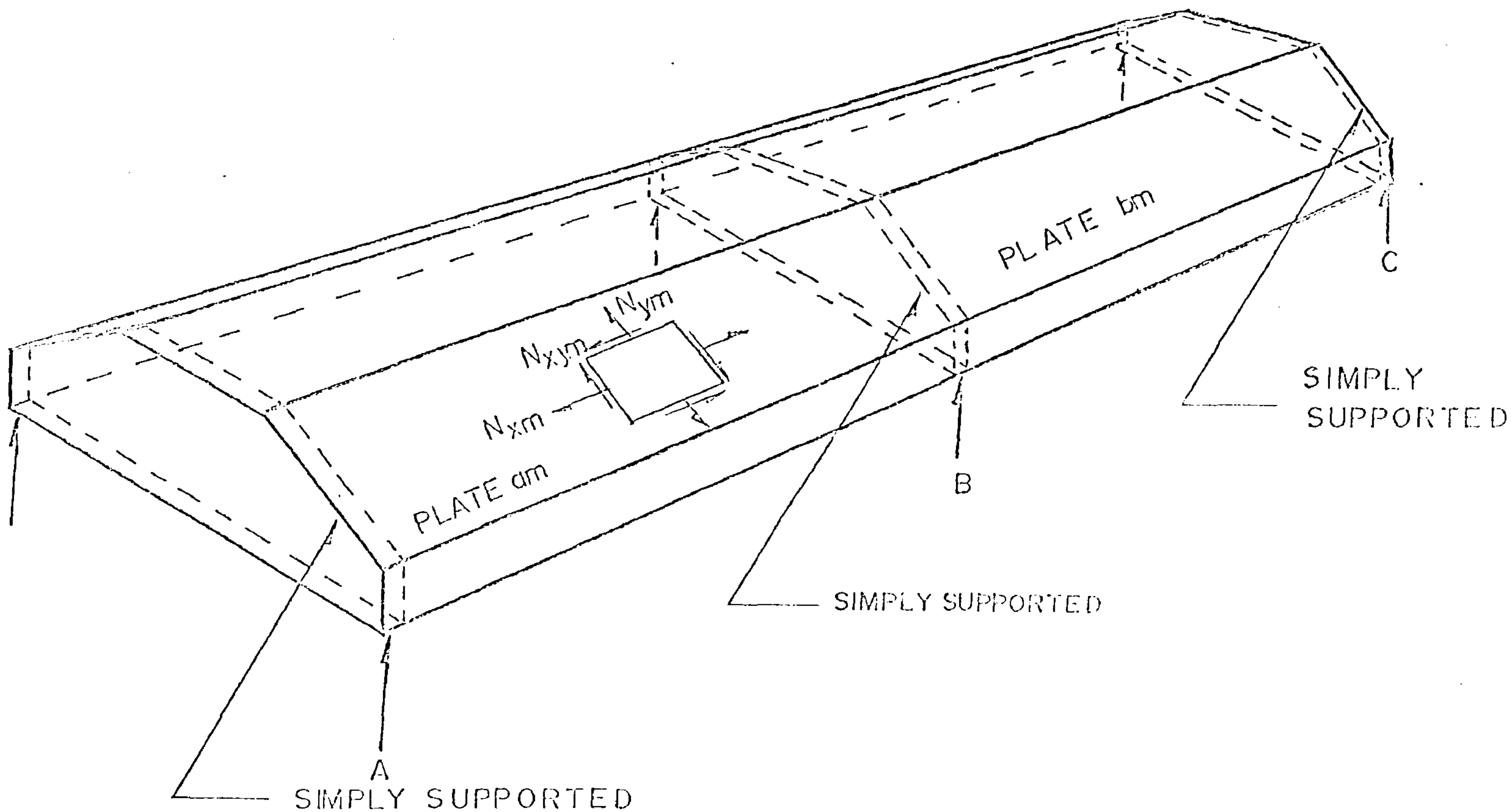


FIG. I. CONTINUOUS FOLDED PLATE STRUCTURE and hangars as well as for the sides and bottoms of elevated bunkers.

The methods of analysis of folded plate structures may fall in the following four categories:

(a) Beam method,

(b) Folded plate theory neglecting relative joint displacement,

(c) Folded plate theory considering relative joint displacement, and

(d) Elasticity method.

Within each of these categories exist many papers using different assumptions and procedures. The term relative joint displacement denotes the displacement of one longitudinal side of a plate with respect to the other longitudinal side.

The basic assumptions in the above methods are the following:

1) The material is homogeneous and linearly elastic.

2) The actual deflections are relatively small as compared to the overall configuration of the structure. Hence, equilibrium conditions for the loaded structure may be developed using the configuration of the undeflected structure.

3) The principle of super-position holds.

4) Longitudinal joints are fully monolithic with slab acting continuously through the joints.

5) Each supporting diaphragm is infinitely stiff parallel to its own plane but is perfectly flexible normal to its plane.

Beam Method.--In this method the structure is assumed to deflect in such a manner that all points in the same cross sections deflect the same amount. Cross sections, however, tend to spread out considerably (Fig. 57) when the folded plate structure is loaded. The beam theory may be applied when the folded plate structures are stiffened with intermediate transverse ribs spaced and designed so as to maintain the shape of the structure.

Folded Plate Theory (with and without relative joint displacement).-- In both classes of folded plate theory the longitudinal supporting action of each plate is governed by beam theory and the transverse supporting action is that of a continuous one-way slab. When the beam theory in each plate is adopted the following assumptions are implied:

1) Longitudinal stresses in each plate vary linearly across the width of each plate but the transverse rate of variation of stress may be different in the various plates.

2) Membrane shearing stresses in each plate have negligible effect on the deflection of the structure.

3) Normal stresses in each plate in the transverse direction are included in equilibrium considerations but have negligible effect on the deflection of the structure.

By adopting the one-way slab action the following assumptions are implied:

4) Slab bending is essentially one way phenomenon occurring in the transverse direction; the effect of longitudinal slab bending is negligible.

5) Individual plates possess negligible torsional resistance. Torsional stresses due to twisting of the plates as well as deflections due to these stresses may be ignored.

6) Radial shearing stresses (normal to the slab) have negligible

effect on the deflection of the structure.

For normal span length to plate width ratios assumptions 1 and 2 do not introduce any substantial inaccuracies. As the span-width ratios become smaller, however, the longitudinal stress becomes non linear and the shear deformation becomes more pronounced. Assumptions 4, 5 and 6 deriving from the use of one-way slab theory are sufficiently accurate for length-width ratios greater than 3 and width-thickness ratios greater than 5 (10).

Folded Plate Theory Neglecting Relative Joint Displacements.-- In this theory it is assumed that the changes in transverse bending moment and in longitudinal stress due to relative joint displacement are negligible in comparison with the values of these moments and stresses computed on the basis of no relative joint displacements. Investigations (10) have shown, however, that joint displacements are too significant to be neglected and must be considered in analyzing folded plate structures.

Folded Plate Theory Considering Relative Joint Displacements.-- This theory is based on the general assumptions which have been previously described, and takes into account the effect of relative displacement of the joints on the transverse moments and membrane stresses. Some of the practical methods of analysis are mentioned below.

1) Vlassow's Method (28, 27). This method determines the values of the stresses at critical sections in the structure by the solution of a set of simultaneous linear algebraic equations that are established on the basis of equilibrium at the critical section and on the basis of continuity of joints transversely. Longitudinal variation of applied loads and transverse moments

are approximated on the basis of a Fourier series.

2) Portland Cement Association Bulletin (24). The analysis contained in this bulletin is similar to that of Vlassow. The typical equations involving the unknown stresses and moments are derived for the case of uniform vertical loading on a simple span structure. The bulletin contains a table of design coefficients for the V-type folded plate structure. Extension of the development to other types of loadings and support conditions is not covered although it is examined in a general way.

3) Gaafar's Method (10). Gaafar used the principle of super position to consider a folded plate structure actually loaded between and at the ridges as the combination of (a) an identical structure loaded between and at the ridges but supported at the ridges against deflection and (b) an identical structure loaded at the ridges only by the ridge reactions of the structure in (a). In developing the theory for the folded plate structure (b), loaded at the ridges only, he makes the assumption that the elastic curve of deflection due to applied load is the same as the normal deflection curve of the structure (half sine wave for simply supported structure). This assumption is accurate within approximately 2% for longitudinally symmetrical loadings. The half sine wave concept as the deflection curve is in error for loading that is largely anti-symmetric longitudinally.

4) Yitzhaki's Method (31). Yitzhaki presents several methods of analysis with major emphasis on the so-called method of particular loadings. In this method he uses the principle of superposition used by Gaafar to analyze the structure for the actual load on it, assuming unyielding ridges. The reactions

developed by these assumed but actually non-existent supports are then applied as longitudinal ridge loads which are carried at each ridge by the plates meeting at each ridge. The resulting deflections of the plates are converted, on the basis of geometrical considerations, into relative joint displacements which are associated with tranverse bending moments in the slabs. From these, a set of ridge loads, called superflous loads, necessary to maintain the deflected shape of the structure is obtained. The structure is then subjected in turn to several appropriately chosen, independent ridge loadings. These loadings are each multiplied by an undetermined constant and combined so as to eliminate the preceding superfluous loading at each ridge. In this process n-2 simultaneous equations each containing the n-2 undetermined constants are obtained and solved to evaluate the constants. Here, n equals the number of plates. With the constants known, the controlling stresses and deflections are readily obtainable. The assumptions and approximations used by Yitzhaki are the same as those used by Gaafar, previously described. Consequently, Yitzhaki's usual procedure is subject to the same limitations on applicability as Gaafar's theory.

Elasticity Method.--Goldberg and Leve developed (13) a solution for the stresses in a folded plate structure by combining the equations of the plate theory for loads normal to the plane of the plates and the elasticity equations defining the plane stress problem for loads in the plane of the plates. Applied loading is approximated by a Fourier series. This method is applied where component plates of the structure are relatively short compared to the width, and where translation of individual joints is completely prevented.

The theory presented in this paper for continuous folded plate structures is developed from Flügge's theory for simply supported folded plate structures which he outlines in his book published in 1962. His theory is particularly attractive since the deflections of plates are given by a differential equation which can be generalised for any loading condition by expressing the load in a Fourier series. Flügge's solution of joint displacement is very much similar to that of Yitzhaki. Chapter VIII of this paper presents the method in detail and extends it to continuous structures, while Chapter III describes the method of analysis by which Flügge's theory is extended to a continuous structure. Chapter XI gives the experimental results of a continuous concrete folded plate structure subjected to uniform load.

CHAPTER II

HISTORICAL DISCUSSION

G. Ehlers (6) wrote one of the first papers on folded plate theory in 1930. Ehlers proposed a folded-plate theory based on linear variation of longitudinal stress in each plate but neglected the effect of the relative displacement of the joints. In 1932 E. Gruber (14) took approximate account of joint translations and rotations by using a strip theory approach. A further refinement of the membrane theory was made by H. Craemer (2) and Gruber (15) by considering the actual stress distributions in the planes of the slab. In 1947 the method was introduced in the U.S.A. in a paper by George Winter, F. ASCE and M. Pei (30). Their theory neglected the relative joint displacements but developed a convenient iteration procedure to determine the logitudinal stresses patterned after the moment distribution procedure.

The relative displacement of the joints which was first proposed by Gruber and Gruening in 1932 (14, 17) was considerably simplified by W. Z. Vlassow, 1936 (28) by using linear algebraic equations to calculate the longitudinal stresses and ridge moments at critical sections of the structure. K. Girkmann and The Portland Cement Association (12, 24) submitted methods similar to Vlassow's. The theory involving relative joint displacements was also developed by I. Gaafar, M. ASCE and D. Yitzhaki (21, 31). A. Werfel and then J. E. Goldberg and H. L. Leve (29, 13) developed approaches that considered both two-dimensional elasticity theory for determination of membrane stresses and two way slab theory for determination of bending and twisting of the slab. In 1962 Flügge published his book in which he outlined the theory which was used in this paper to develop a method to analyze continuous folded plate structures.

CHAPTER III

METHOD OF ANALYSIS

Plate strip m, bounded by edges m-1 and m (Fig. 1) and subjected to the resolved uniform loads S_{am} and S_{bm}, is analyzed as a continuous beam supported at A, B, and C. The continuous beam AC is then divided into two simply supported beams AB and BC (Fig. 2b). Due to the uniform load S_{bm} plate BC rotates θ_{bm} degrees at end B, while plate AB, due to the uniform load S_{am}, rotates $-\theta_{am}$ degrees at end B (Fig. 2c). Similarly deflections v_{bm} and v_{am} are produced.

The relative rotation of tangents a and b is $\theta_b - \theta_c$. In the continuous plate AC, however, the tangents a and b form a straight line since the deflection curve must be continuous. It is, therefore, necessary to apply a set of moments M^r_{om} at ends B of plates AB and BC which will produce a relative rotation equal and opposite to $\theta_{bm} - \theta_{am}$, where r in M^r_{om} goes from K = 1 to K - 1 and K = number of plates. The stresses and deflections due to the moment M_{om} would then be added to the stresses and deflections produced by the uniform loads S_{am} and S_{bm}. The resulting stresses and deflections would be those of the continuous plate AC subjected to loads S_{am} and S_{bm} at spans AB and BC respectively. The moment M_{om}, however, is an unknown quantity and must be found.

At ends B of plates AB and BC a unit moment is applied which will produce rotations θ_{cm}, θ_{dm} and deflections v'_{cm} and v'_{dm} (Fig. 2d). The relative rotation between tangents d and c is $\theta_{dm} - \theta_{cm}$. Hence

$$M^{r}_{om} = \frac{\theta_{am} - \theta_{bm}}{\theta_{cm} - \theta_{dm}} . \tag{3-1}$$

The stresses and deflections of the continuous plate can then be computed. Once these stresses and deflections have been obtained joint displacements can be considered as described in Chapter IX.

Computation of S_m

It is assumed that the edge loads are vertical and uniformly distributed in the x direction. The load at every edge may be different in magnitude but it must be uniformly distributed along x throughout the whole length of the beam.

Flügge (8) resolves the load P_m acting on an element of unit length of the m-th edge into two components in the directions y_m and y_{m+1}:

$$S'_m = - P_m \frac{\cos \phi_{m+1}}{\sin \gamma_m} , \quad S''_m = + P_m \frac{\cos \phi_m}{\sin \gamma_m} \tag{3-2}$$

These loads can be carried by the strips m and m + 1 respectively (Fig. 3c). The resultant load acting on plate m would be

$$S_m = S''_{m-1} + S'_m . \tag{3-3}$$

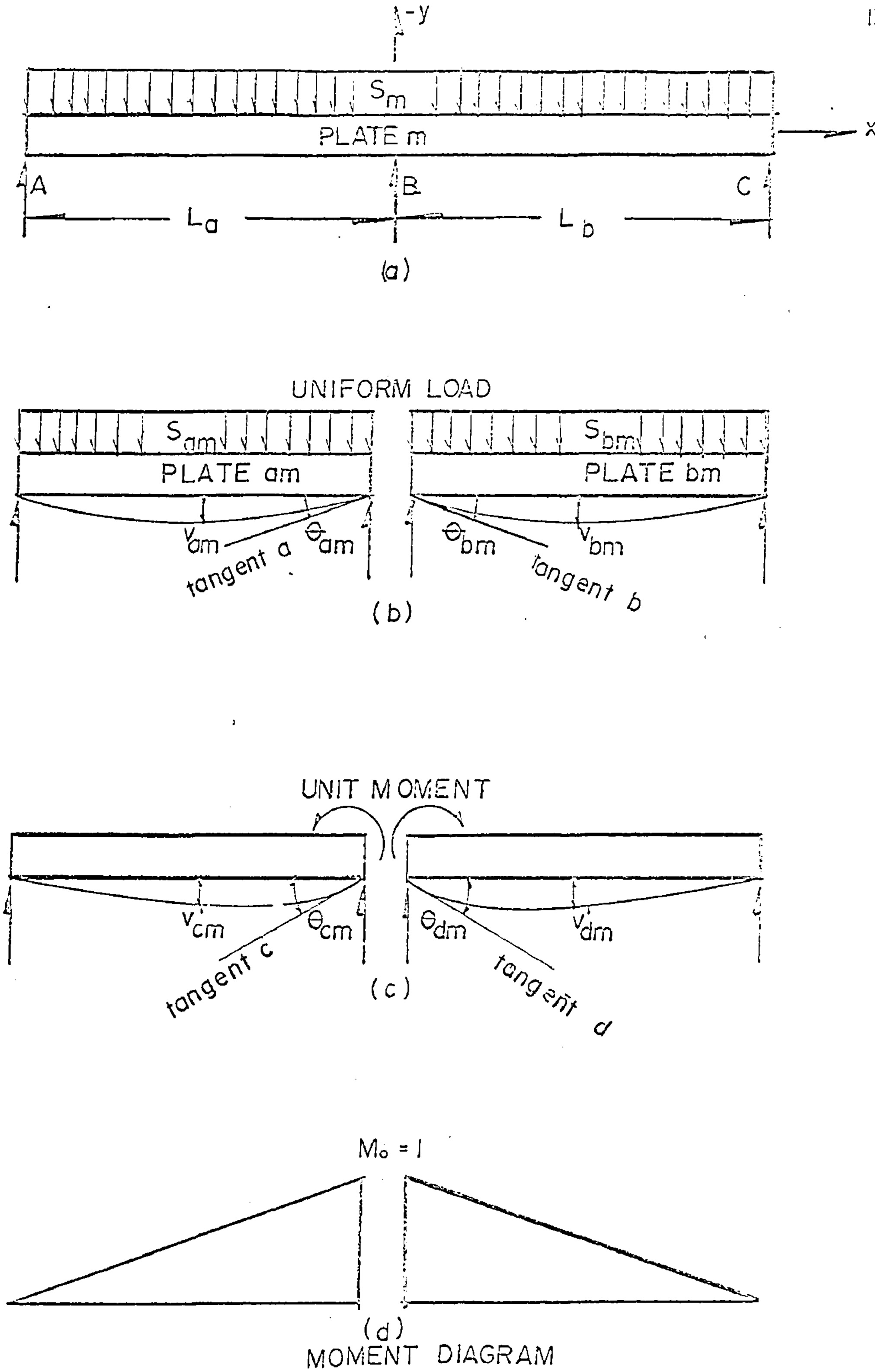


FIG. 2. METHOD OF ANALYSIS

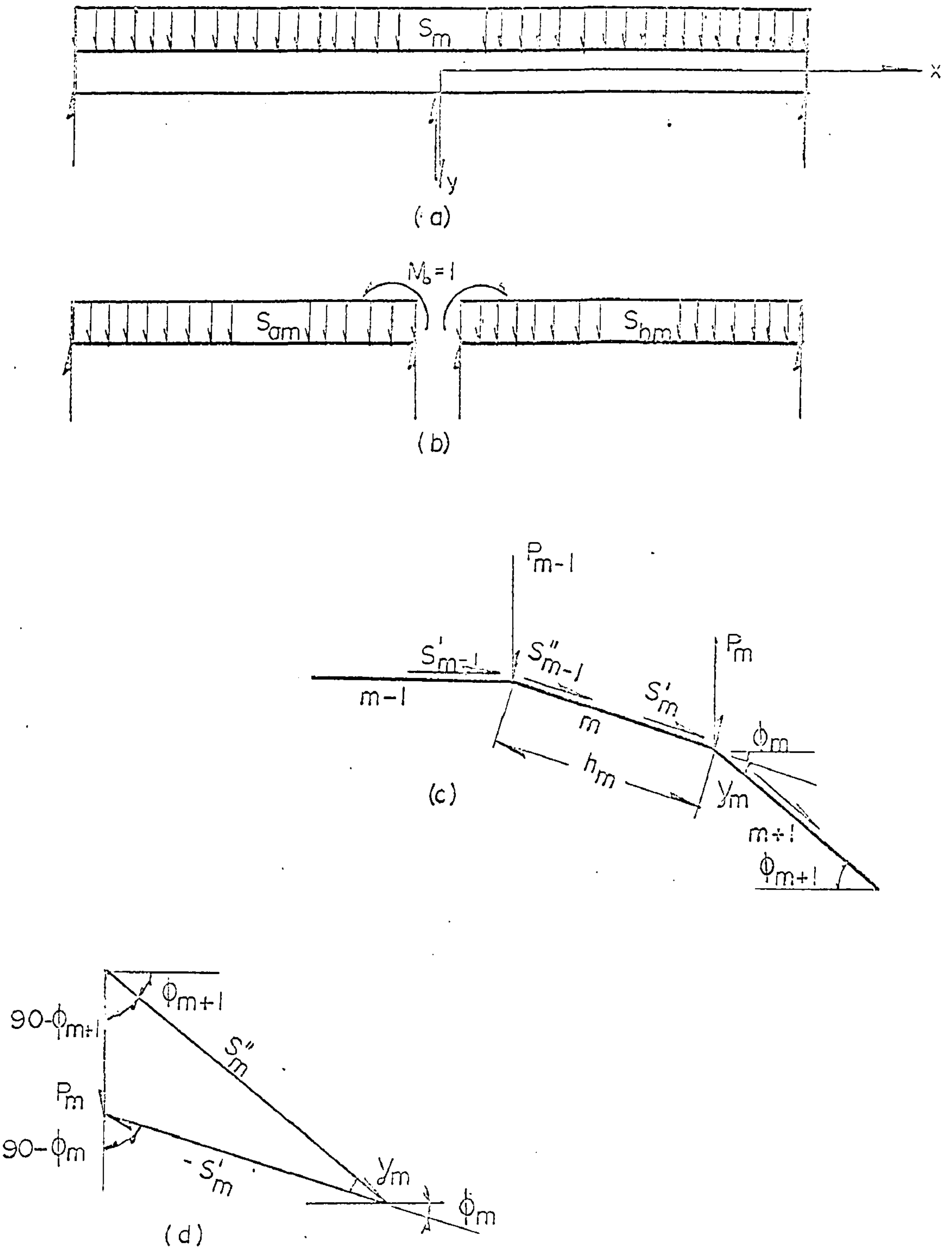


FIG.3. THE UNIFORM LOAD S_m IS RESOLVED INTO FORCES ACTING ON THE PLANES OF PLATES

CHAPTER IV

ANALYSIS OF PLATE $_{bm}$ UNDER UNIFORM LOAD S_{bm}

Bending Moment Due to S_{bm}

The beam shown in Fig. 4 (a) subjected to the uniform load S_{bm} has the bending moment

$$M^{(0)}_{bm} = S_{bm} \frac{L_b x - x^2}{2} \tag{4-1}$$

Shearing Forces Due to S_{bm}

$$Q^{(0)}_{bm} = S_{bm} \left(\frac{L_b}{2} - x \right). \tag{4-2}$$

Let us now assume that the plate strip is slender, that is, the height h_m is much smaller than the length L_b, then the bending stress σ_x and the shear stress τ may be found from formulas of elementary beam theory. Moreover, let $N_x = \sigma_x t_m$ and $N_{xy} = \tau t_m$ where t_m is the thickness of the plate (Fig. 5). Hence

$$N^{(0)}_{bxy} = \tau t_m = \frac{VA\bar{y}}{I} = \frac{d M^{(o)}_{bm}}{dx} \frac{A\bar{y}}{I} \tag{4-3}$$

$$N^{(0)}_{bxym} = \frac{Q^{(0)}_{bm} \left(\frac{h_m}{2} - y_m \right) \left[y_m + \frac{1}{2} \left(\frac{h_m}{2} - y_m \right) \right] t_m}{\frac{1}{12} t_m h_m^3} \tag{4-4}$$

Simplifying the above equation

$$N^{(0)}_{bxy_m} = \frac{6Q^{(0)}_{bm}}{h_m} \left(\frac{1}{4} - \frac{y_m^2}{h_m^2} \right) . \tag{4-5}$$

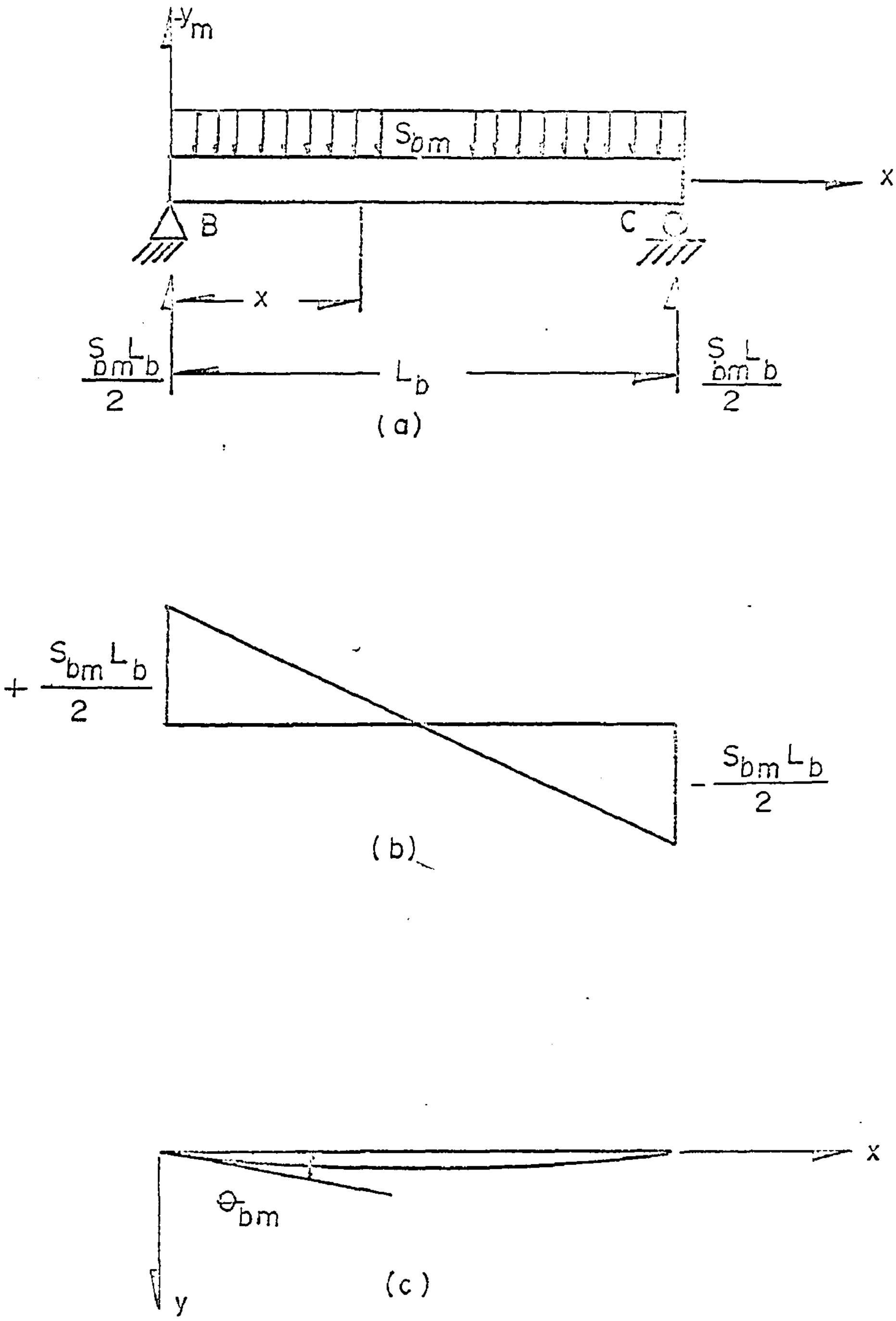


FIG. 4. PLATE bm UNDER UNIFORM LOAD S_{bm}

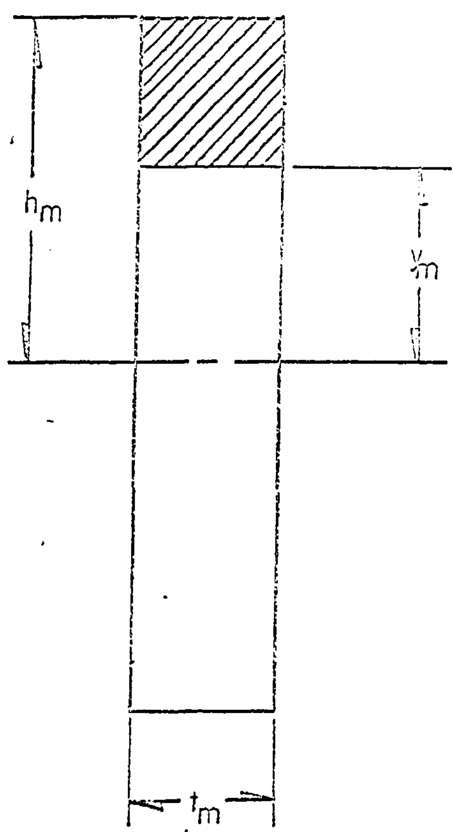


FIG. 5. PLATE CROSS-SECTION

Also the stress

$$N^{(0)}_{bxm} = \sigma_x t_m = \frac{12 M^{(0)}_{bm} y_m}{h^3_m} \qquad (4-6)$$

Longitudinal Shear Stress Equations

At the lower edge of the strip m ($y_m = + \frac{h_m}{2}$) the normal force $N^{(0)}_{bxm}$ produces the strain

$$\epsilon^{(0)}_{bxm} = \frac{N^{(0)}_x}{E t_m} = \frac{6 M^{(0)}_m}{E t_m h^2_m} \qquad (4-7)$$

And the strain at the upper edge of the adjacent strip is

$$\epsilon^{(0)}_{b(x+1)m} = - \frac{6 M^{(o)}_{m+1}}{E t_{m+1} h^2_{m+1}} \qquad (4-8)$$

Since the two adjacent plates are connected to one another these strains must be made equal to each other. When this operation is done the two strips will exert forces upon each other. These additional forces are shearing forces T_m acting on the plane of both strips. These forces are shown in Fig. 7 acting in a positive direction in accordance with the sign convention for N_{xy} (Fig. 6b).

From Fig. 7 it is seen that the edge shears will add an additional moment $M^{(1)}_m$ to the bending moment $M^{(0)}_m$ of the strip. An axial force $N^{(1)}_m$ is also produced. Applying the equilibrium equations to the strip element shown in Fig. 7 the following relations are obtained.

$$\frac{dN^{(1)}_{bm}}{dx} = T_{bm-1} - T_{bm} \qquad (4-9)$$

$$\frac{dM^{(1)}_{bm}}{dx} = - \frac{h_m}{2} (T_{bm-1} + T_{bm}) \qquad (4-10)$$

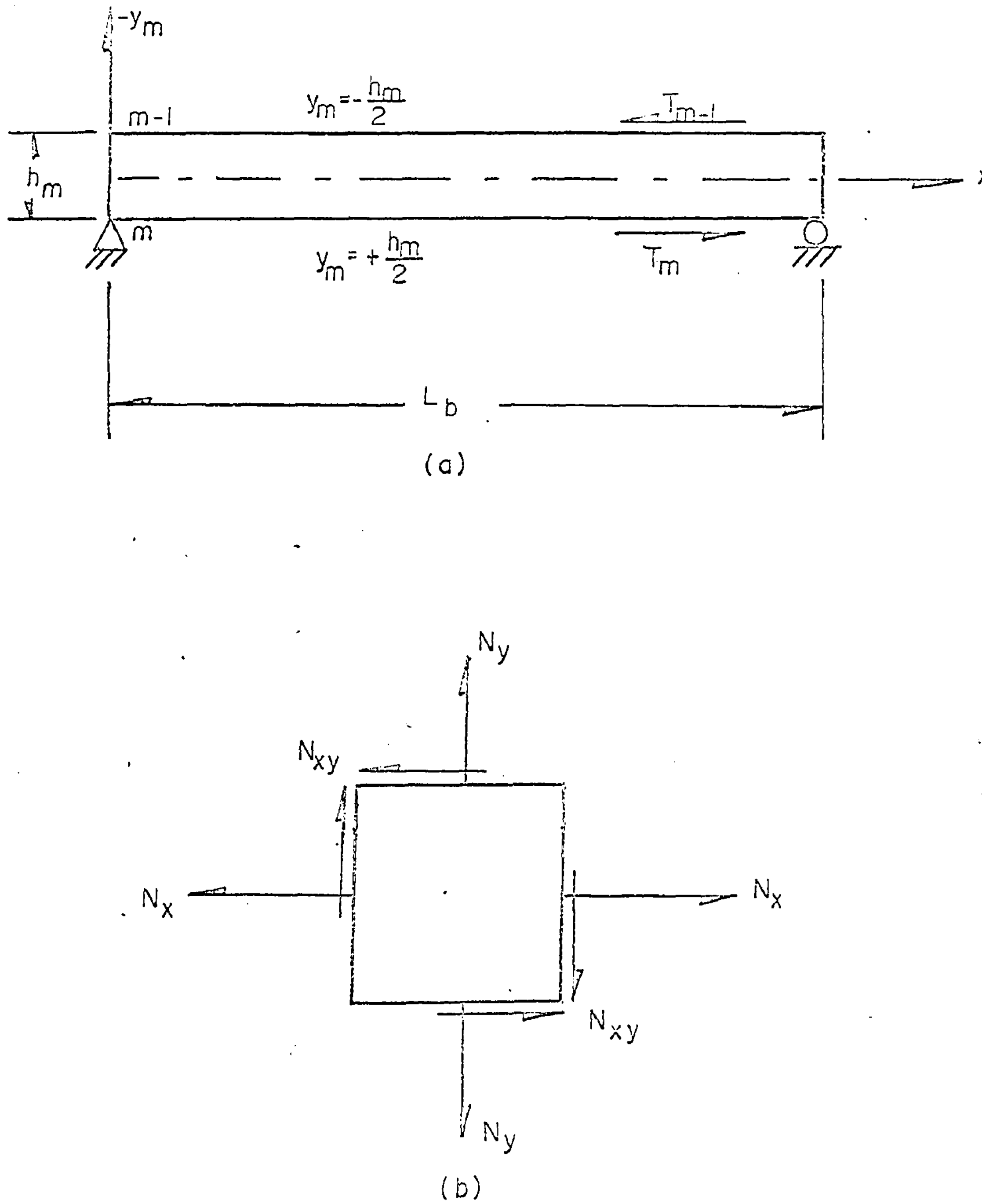


FIG. 6. LONGITUDINA SHEARS IN PL bm DUE TO UNIFORM LOAD S_{bm}

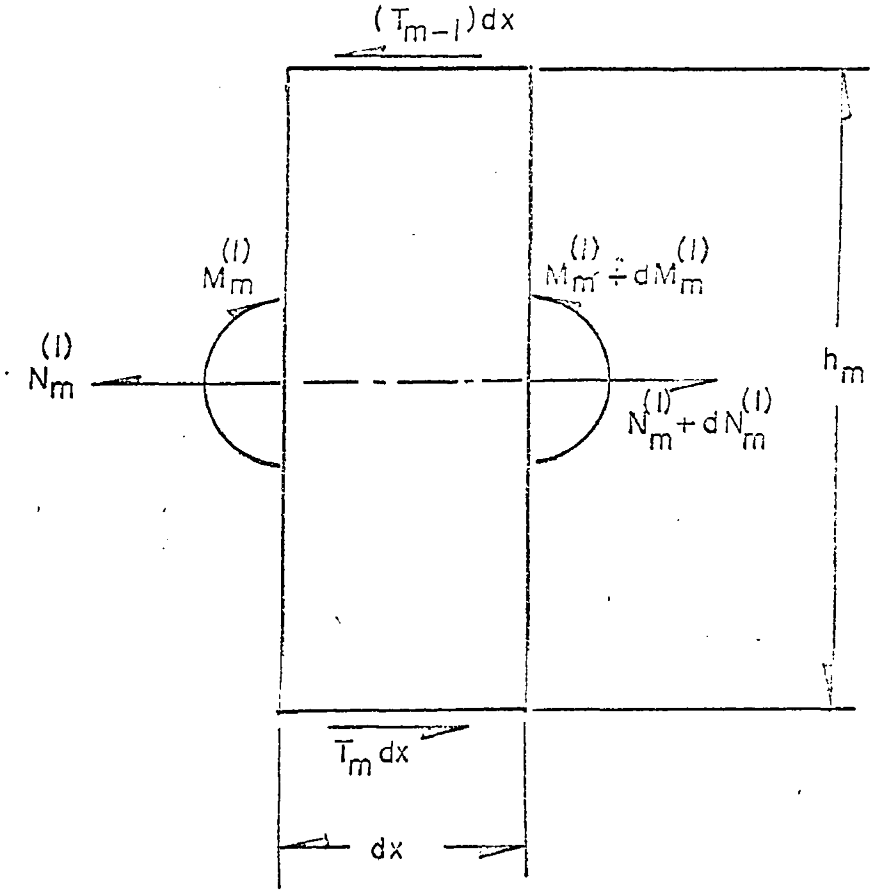


FIG. 7. STRIP ELEMENT OF R bm

The shearing forces T_{bm} must be known in terms of x before (4-9) and (4-10) can be integrated.

The shearing forces T_m produce the force $N_{bm}^{(1)}$ and the moment $M_{bm}^{(1)}$ which in turn produce beam bending stresses from which the normal force

$$N_{bx}^{(1)} = \frac{N_{bm}^{(1)}}{h_m} + \frac{12\, M_{bm}^{(1)}\, y_m}{h_m^3} \qquad (4\text{-}11)$$

and the edge strain

$$\epsilon_{bm}^{(1)} = \frac{N_{bm}^{(1)}}{Et_m\, h_m} \pm \frac{M_{bm}^{(1)}}{Et_m\, h_m^2} \qquad (4\text{-}12)$$

In equation (4-12) the second term on the right side is negative if $y = -\frac{h_m}{2}$ and positive if $y = +\frac{h_m}{2}$.

To avoid discrepancy of strains between connecting edges the following condition must exist.

$$\epsilon_{bx\,m}^{(0)} + \epsilon_{bx\,m}^{(1)} = \epsilon_{bx(m+1)}^{(0)} + \epsilon_{bx(m+1)}^{(1)} \qquad (4\text{-}13)$$

After having substituted expressions for the ϵ's

$$\frac{6\, M_{bm}^{(0)}}{E\, t_m\, h_m^2} + \frac{N_{bm}^{(1)}}{E\, t_m\, h_m} + \frac{6\, M_{bm}^{(1)}}{E\, t_m\, h_m^2} = -\frac{6M_{b(m+1)}^{(0)}}{E\, t_{m+1}\, h_{m+1}^2} + \frac{N_{b(m+1)}^{(1)}}{E\, t_{m+1}\, h_{m+1}} - \frac{6\, M_{b(m+1)}^{(1)}}{E\, t_{m+1}\, h_{m+1}^2} \qquad (4\text{-}14)$$

Equation (4-14) is the compatibility equation from which an equation for the unknown shearing forces T_m can be derived. After differentiation in respect to x, equation (4-14) yields

$$6\,\frac{\dfrac{d\,M^{(0)}_{bm}}{dx}}{E\,t_m\,h_m^2} + \frac{\dfrac{d\,N^{(1)}_{bm}}{dx}}{E\,t_m\,h_m} + 6\,\frac{\dfrac{d\,M^{(1)}_{bm}}{dx}}{E\,t_m\,h_m^2} =$$

$$-\,\frac{6\,\dfrac{d\,M^{(o)}_{b(m+1)}}{dx}}{E\,t_{m+1}\,h^2_{m+1}} + \frac{\dfrac{d\,N^{(1)}_{b(m+1)}}{dx}}{E\,t_{m+1}\,h_{m+1}} - \frac{6\,\dfrac{d\,M^{(1)}_{b(m+1)}}{dx}}{E\,t_{m+1}\,h^2_{m+1}} \qquad (4\text{-}15)$$

Equation (4-1) is differentiated in respect to x to yield

$$\frac{d\,M^{(0)}_{bm}}{dx} = \frac{S_{bm}}{2}(L_b - 2\,x). \qquad (4\text{-}16)$$

Equations (4-16), (4-9), and (4-10) are substituted in equation (4-15). After simplification

$$\frac{1}{t_m\,h_m}\left(2T_{b(m-1)} + 4T_{bm}\right) + \frac{1}{t_{m+1}\,h_{m+1}}\left(4T_{bm} + 2T_{b(m+1)}\right)$$

$$= \frac{3S_{bm}(L_b - 2\,x)}{t_m\,h_m^2} + \frac{3S_{b(m+1)}(L_b - 2\,x)}{5\,t_{m+1}\,h^2_{m+1}} \qquad (4\text{-}17)$$

A total of (K-1) equations such as (4-17) may be written for every edge from m = 1 to m = K-1 in order to solve K - 1 unknown functions T_m (x). Since in equation (4-17) both terms on the right hand side are proportional to (L_b - 2x), it may be said that

$$\begin{aligned} T_{bm} &= T'_{bm}\ (L_b - 2\,x) \\ T_{b(m+1)} &= T'_{b(m+1)}\ (L_b - 2\,x) \end{aligned} \qquad (4\text{-}18)$$

where T'_m, T'_{m+1} . . . . . are constants.

Expressions (4-18) are substituted in equations (4-17). The resulting equations are then differentiated in respect to x and simplified to yield a set of ordinary linear equations for the unknown constants T'_m.

$$\frac{1}{t_m\, h_m}\, T'_{b(m-1)} + 2\left(\frac{1}{t_m\, h_m} + \frac{1}{t_{m+1}\, h_{m+1}}\right) T'_{bm} + \frac{1}{t_{m+1}\, h_{m+1}}\, T'_{b(m+1)}$$

$$= \frac{\frac{3 S_{bm}}{2}}{t_m\, h_m} + \frac{\frac{3 S_{b(m+1)}}{2}}{t_{m+1}\, h_{m+1}} \qquad (4\text{-}19)$$

<u>The Bending Moment Resultant M_{bm}</u>

The moment of the strip due to the uniform load S_{bm} and the shearing force T_m may be expressed as

$$M_{bm} = M^{(0)}_{bm} + M^{(1)}_{bm} \qquad (4\text{-}20)$$

The moment $M^{(1)}_m$ is found by substituting expressions (4-18) into equation (4-10) and integrating in respect to x. Hence

$$\frac{d\, M^{(1)}_{bm}}{d x} = - \frac{h_m}{2} \left(T^1_{bm-1} + T^1_{bm}\right) (L_b - 2\, x) \qquad (4\text{-}21)$$

$$M^{(1)}_{bm} = - \frac{h_m}{2} \left(T^1_{b(m-1)} + T^1_{bm}\right) (L_b x - x^2) + C \qquad (4\text{-}22)$$

Substituting the boundary conditions

$$x = 0 \qquad M^{(1)}_{bm} = 0$$

$$x = L_b \qquad M^{(1)}_{bm} = 0$$

into equation (4-22) it is found that C = 0. Therefore

$$M^{(1)}_{bm} = - \frac{h_m}{2} \left(T^1_{bm-1} + T^1_{bm}\right) (L_b\, x - x^2)$$

but $\qquad M_{bm} = M^{(0)}_{bm} + M^{(1)}_{bm}$

Hence

$$M_{bm} = \frac{S_{bm}}{2}(L_b x - x^2) + \frac{b_m}{2}(T^1_{m-1} + T^1_m)(x^2 - L_b x) \tag{4-23}$$

The Shear Stress Resultant Q_{bm}

The total shear stress resultant $Q_m = Q_m^{(0)} + Q_m^{(1)}$. But $Q_m^{(1)} = O$ since the shears T_m don't produce any vertical shear component. Therefore

$$Q_{bm} = \frac{S_{bm} L_b}{2} - S_{bm} x$$

$$Q_{bm} = S_{bm} \left(\frac{L_b}{2} - x \right) \tag{4-24}$$

The Normal Stress Resultant N_{bm}

The total normal stress resultant $N_{bm} = N_{bm}^{(0)} + N_{bm}^{(1)}$. But $N_{bm}^{(0)} = O$ since the uniform load S_{bm} does not produce any net force in the x-direction at a given cross section. The $N_{bm} = N_{bm}^{(1)}$. Equation (3-10) states that

$$\frac{d N^1_{bm}}{dx} = T_{b(m-1)} - T_{bm}$$

$$\frac{d N^{(1)}_{bm}}{dx} = (T^1_{b(m-1)} - T^1_{bm})(L_b - 2x)$$

$$N^{(1)}_{bm} = (T^1_{bm-1} - T^1_{bm})(L_b x - x^2) + C \tag{4-25}$$

at $x = O$ and at $x = L_b$ $N_m^{(1)} = O$. Therfore $C = O$.

$$N_{bm} = (T^1_{b(m-1)} - T^1_{bm})(L_b x - x^2) \tag{4-26}$$

The Stress Resultant N_{bx}

From elementary theory

$$\sigma_x = \frac{P}{A} \pm \frac{My}{I}$$

Then

$$\sigma_{bx} = \frac{(T^1_{b(m-1)} - T^1_{bm})(L_2 x - x^2)}{t_m h_m} + \frac{12 y_m}{t_m h_m^3}\left[\frac{S_{bm}}{2}(L_2 x - x^2) + (T^1_{b(m-1)} + T^1_{bm})\left(\frac{h_m}{2}\right)(x^2 - L_b x)\right] \quad \text{but} \; N_{bx} = t_m \, \sigma_{bx} \tag{4-27}$$

$$N_{bx} = \frac{L_b x - x^2}{h_m}\left[-(T_{b(m-1)} - T^1_{bm}) + \frac{6 \, S_{bm} \, y_m}{h_m^2} - \frac{6 \, y_m}{h_m}(T^1_{b(m-1)} + T^1_{bm})\right] \tag{4-28}$$

Shear Stress Resultant N_{bxy}

$$N_{bxy} = N^{(0)}_{bxy} + N^{(1)}_{bxy} \tag{4-29}$$

$$N^{(0)}_{bxy} = \tau^{(0)}_b \, t_m = \frac{\frac{d M^{(0)}_{bm}}{dx} Q}{I \, m}$$

$$I = \frac{1}{12} t_m h_m^3$$

$$Q = A\bar{y} = \frac{t_m}{8}(h_m^2 - 4 y_m^2)$$

$$N^{(0)}_{bxy} = \frac{3}{4} \frac{S_{bm}}{h_m^3}(L_b - 2x)(h_m^2 - 4 y_m^2) \tag{4-30}$$

To find $N^{(1)}_{xy}$ the element shown in Fig. 9 is considered.

$$\tau^{(1)}_b \, t_m \, dx = T^1_{bm}(L_b - 2x)\, dx + \int_{y_m}^{\frac{h_m}{2}} d N^{(1)1}_{bx} \, dy \tag{4-31}$$

$$N^{(1)}_{bx} = \frac{N^{(1)}_{bm}}{h_m} + \frac{12 M^{(1)}_{bm} \, y_m}{h_m^3}$$

Differentiating $N^{(1)}_x$ in respect to x and substituting for $\frac{d N^{(1)}_{bm}}{dx}$ and $\frac{d M^{(1)}_{bm}}{dx}$,

the following equation is obtained.

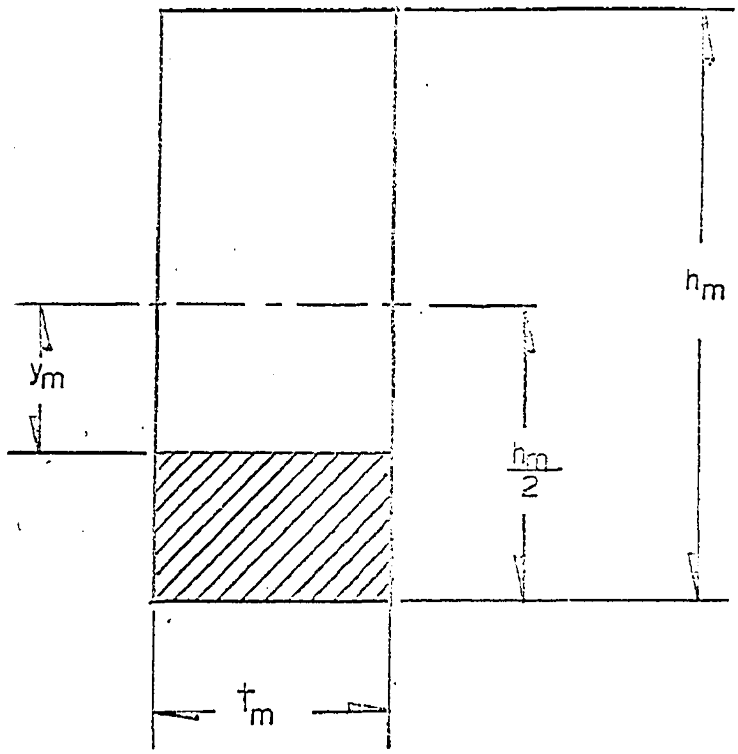


FIG. 8. MOMENT AREA OF CROSS-SECTION

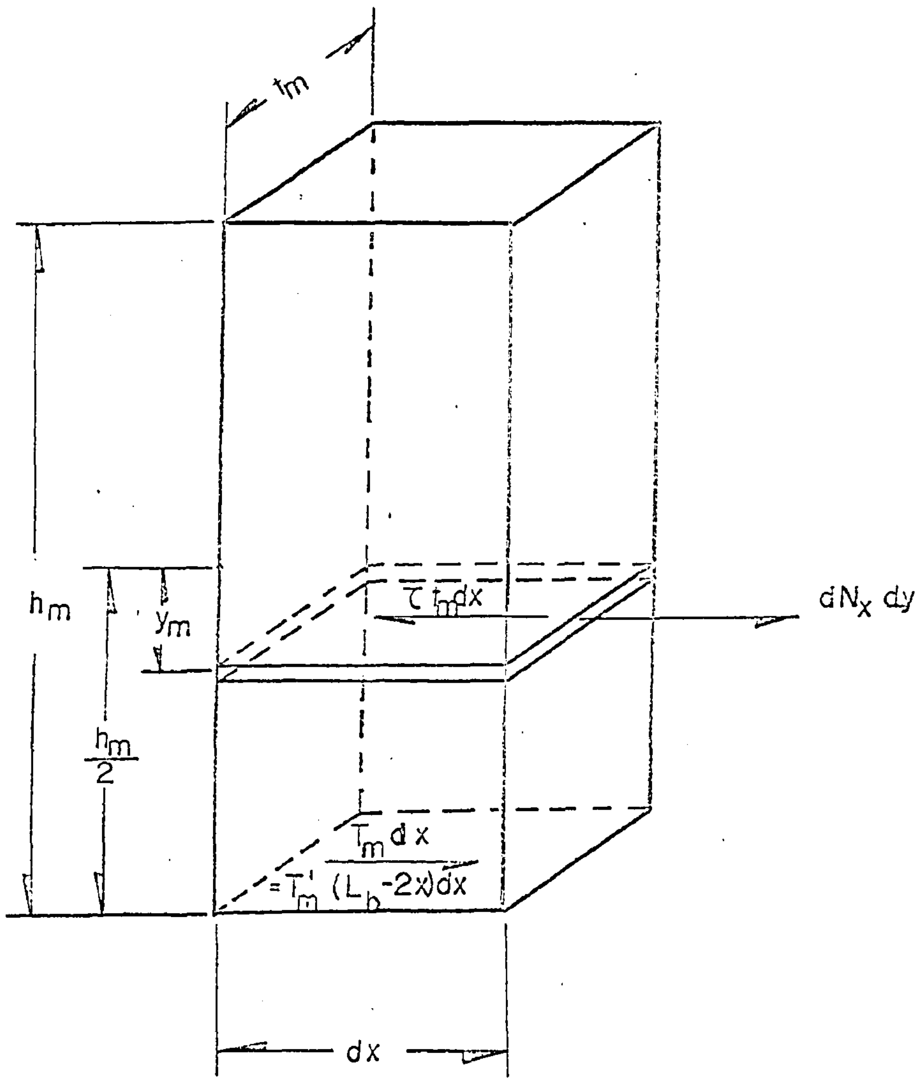


FIG. 9. VOLUME ELEMENT OF PLATE bm
UNDER LOAD S_{bm}

$$d N_{bx}^{(1)} = \frac{(L_b - 2x)}{h_m} (T^1_{b(m-1)} - T^1_{bm}) \quad dx$$

$$- \frac{6 (L_b - 2x)}{h_m^2} (T^1_{b(m-1)} + T^1_{bm}) \; y \, dx \qquad (4\text{-}32)$$

Equation (4-32) is substituted into equation (4-31), which is integrated in respect to y and simplified.

$$N^1_{bxy} = \frac{T^1_{bm} (L_b - 2x)}{4h_m^2} (2 y_m + h_m) (6 y_m - h_m)(2 y_m - h_m) (6 y_m + h_m) \qquad (4\text{-}33)$$

Equations (4-33) and 4-30) are substituted into equation (4-29).

$$N_{bxy} = \frac{3}{4} \frac{S_{bm}}{h_m^3} (L_b - 2x)(h_m^2 - 4y_m^2) + \frac{T^1_{b(m-1)} (L_b - 2x)}{4 h_m^2} (2 y_m - h_m) (6 y_m + h_m)$$

$$+ \frac{T^1_{bm} (L_b - 2x)}{4 h_m^2} (2y_m + h_m) (6y_m - h_m) \qquad (4\text{-}34)$$

The normal force N_y in the direction of the y_m axis shown in Fig. 6(b) is rather small and there is no need to compute it.

Rotation of Plate BC

The deflection of plate BC is subject to the differential equation

$$\frac{d^2 v_{bm}}{d x^2} = - \frac{12}{E t_m h_m^3} M_{bm} \qquad (4\text{-}35)$$

The moment M_m is given in terms of T^1_m in equation (3-23) which is substituted in equation (4-35) to yield

$$\frac{d^2 v_{bm}}{d x^2} = \frac{12}{E t_m h_m^3} \left[\frac{b_m}{2} (L_b x - x^2) + \frac{h_m}{2} (T^1_{m-1} + T^1_m) (x^2 - L_b x) \right] \qquad (4\text{-}36)$$

If integrated once in respect to x, equation (4-36) will yield the slope of the elastic curve.

$$\frac{d\,v_{bm}}{d\,x} = \theta_{bx} = \frac{12}{E\,t_m\,h_m^3}\left[\frac{S_{bm}\,L_b\,x^2}{4} - \frac{S_{bm}x^3}{6} + \frac{h_m}{2}\left(T^1_{b(m-1)} + T^1_{bm}\right)\left(\frac{x^3}{3} - \frac{L_b x^2}{2}\right)\right] + C \tag{4-37}$$

$$\text{at } x = \frac{L_b}{2} \qquad \theta = 0. \tag{4-38}$$

The boundary condition (4-38) is substituted in equation (4-37). After simplification

$$C = -\frac{12}{E\,t_m\,h_m^3}\left[\frac{S_{bm}\,L_b^3}{24} - \frac{h_m}{2}\left(T^1_{bm-1} + T^1_{bm}\right)\frac{L_b^3}{12}\right] \tag{4-39}$$

The integration constant (4-39) is substituted into equation (4-37) to obtain the equation of the slope of the elastic curve.

$$\theta_{bx} = \frac{12}{E\,t_m h_m^3}\left[\frac{S_{bm}\,L_b\,x^2}{4} - \frac{S_{bm}\,x^3}{6} + \frac{h_m}{2}\left(T^1_{b(m-1)} + T^1_{bm}\right)\left(\frac{x^3}{3} - \frac{L_b\,x^2}{2}\right)\right]$$

$$-\frac{12}{E\,t_m\,h_m^3}\left[\frac{S_{bm}\,L_b^3}{24} - \frac{h_m}{2}\left(T^1_{m-1} + T^1_m\right)\frac{L_b^3}{12}\right] \tag{4-40}$$

$$\theta_{bx=0} = -\frac{12}{E\,t_m\,h_m^3}\left[\frac{S_{bm}\,L_b^3}{24} - \frac{h_m}{2}\left(T^1_{m-1} + T^1_m\right)\frac{L_b^3}{12}\right] \tag{4-41}$$

Deflection of Plate BC

In order to find the equation of the deflection curve of plate BC, equation (4-36) must be integrated twice in respect to x.

$$v_{bx} = \frac{12}{E\,t_m\,h_m^3}\left[\frac{S_{bm}\,L_b\,x^3}{12} - \frac{S_{bm}\,x^4}{24} + \frac{h_m}{2}\left(T^1_{b(m-1)} + T^1_{bm}\right)\left(\frac{x^4}{12} - \frac{L_b\,x^3}{6}\right)\right]$$

$$+ Cx + D \tag{4-42}$$

The deflection $v_{bxm} = 0$ at $x = 0$ and at $x = L_b$. Applying these conditions to equation (4-42) it is found that the integration constant $D = 0$ and the integration constant C is the same as that given by equation (4-39). The values of C and D are substituted back into (4-42). After simplification

$$v_{bxm} = - \frac{1}{2\, E t_m h_m^3} \left[(x^4 - 2\, L_b\, x^3 + L_b^3\, x) \left(S_{bm} - h_m \left\langle T^1_{bm-1} + T^1_{bm} \right\rangle \right) \right] \quad (4\text{-}43)$$

CHAPTER V

ANALYSIS OF PLATE am UNDER UNIFORM LOAD S_{am}

Plate am shown in Fig. 10(a), is analyzed in the same way as plate bm. Most of the resulting equations are correspondingly the same as those shown for plate bm while others may show some differences in negative and positive signs. For this reason explanations and derivations of equations have been omitted in this section. The reader may refer back to plate bm for more details.

Bending Moment Due to S_{am}

$$M_{am}^{0} = -S_{am} \frac{L_b x + x^2}{2} \qquad (5-1)$$

Shear Forces Due to S_{am}

The reader may refer to equations (4-2) to (4-6) for more details.

$$Q_{am}^{(0)} = -S_{am} \left(\frac{L_a}{2} + x \right) \qquad (5-2)$$

$$N_{axy}^{(0)} = \frac{6\, Q_{am}^{0}}{b_m^{3}} \left(\frac{1}{4} - \frac{y_m^2}{b_m^2} \right) \qquad (5-3)$$

$$N_{ax}^{(0)} = \frac{12\, M_{am}^{(0)}\, y_m}{b_m^{3}} \qquad (5-4)$$

Longitudinal Shear Stress Equations

The main equations used to develop the shear stress equations are

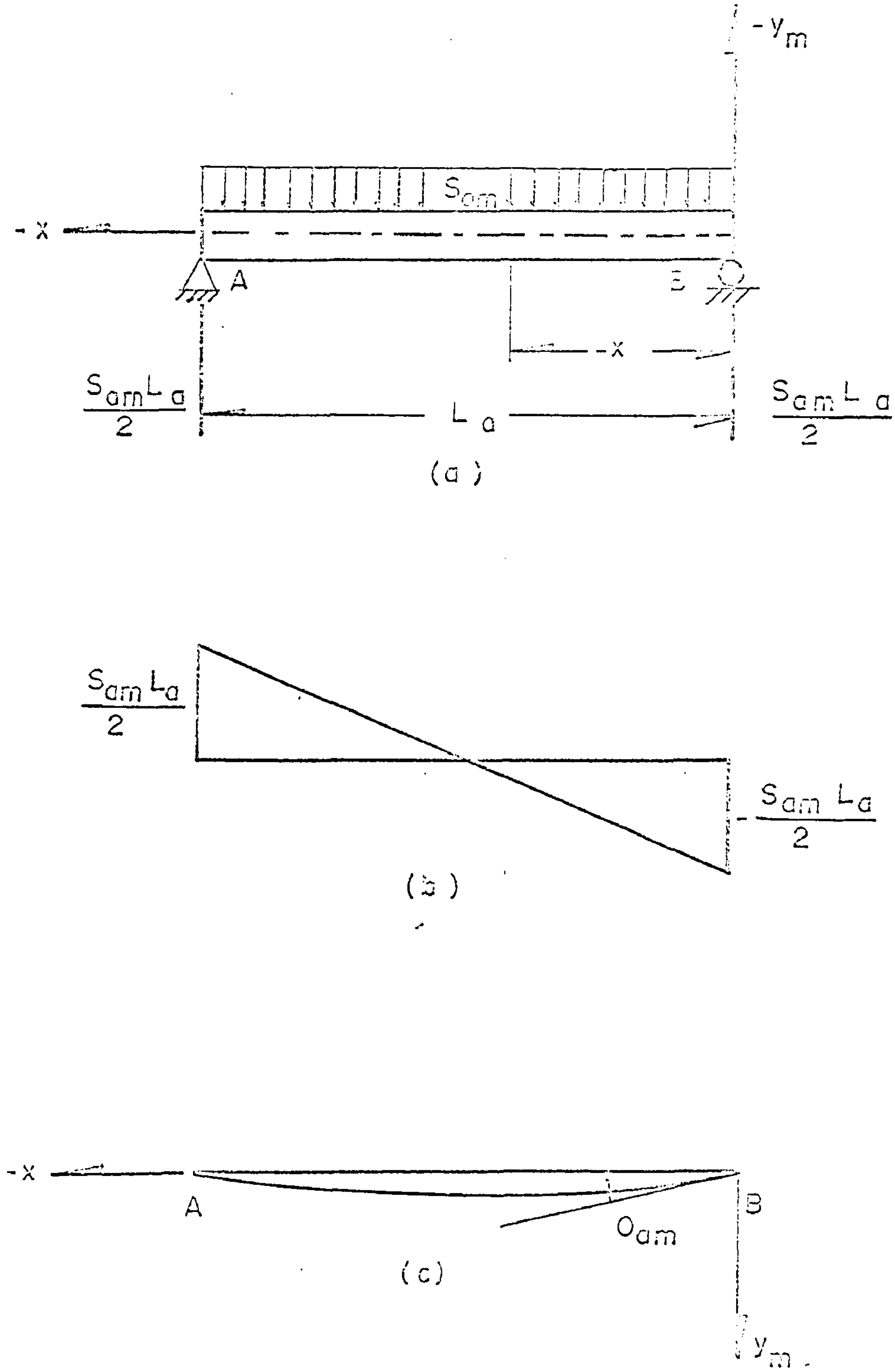


FIG. 10. PLATE am UNDER UNIFORM LOAD S_{am}

shown below. The reader may refer to equations (4-7) to (4-19) for details.

$$\epsilon^{(0)}_{axm} = \frac{6\, M^{(0)}_{am}}{E\, t_m\, h^2_m} \tag{5-5}$$

$$\epsilon^{(0)}_{ax(m+1)} = -\frac{6\, M^{(0)}_{a(m+1)}}{E\, t_{m+1}\, h^2_{m+1}} \tag{5-6}$$

$$\frac{d N^{(1)}_{am}}{dx} = T_{a(m-1)} - T_{am} \tag{5-7}$$

$$\frac{d\, M^{(1)}_{am}}{dx} = -\frac{h_m}{2}\left(T_{(am-1)} + T_{am}\right) \tag{5-8}$$

$$N^{(1)}_{axm} = \frac{N^{(1)}_{am}}{h_m} + \frac{12\, M_{am}\, y_m}{h^3_m} \tag{5-9}$$

$$\epsilon^{(1)}_{axm} = \frac{N^{(1)}_{am}}{E\, t_m\, h_m} \pm \frac{6\, M^{(1)}_{am}}{E\, t_m\, h^2_m} \tag{5-10}$$

$$\epsilon^{(0)}_{axm} + \epsilon^{(1)}_{axm} = \epsilon^{(0)}_{ax(m+1)} + \epsilon^{(1)}_{Aax(m+1)} \tag{5-11}$$

$$\frac{6\, M^{(0)}_{am}}{E\, t_m\, h^2_m} + \frac{N^{(1)}_{am}}{E\, t_m\, h_m} + \frac{6\, M^{(0)}_{am}}{E\, t_m\, h^2_m} =$$

$$-\frac{6\, M^{0}_{a(m+1)}}{E\, t_{m+1}\, h^2_{m+1}} + \frac{N^{(1)}_{a(m+1)}}{E\, t_{m+1}\, h_{m+1}} - \frac{6\, M^{(1)}_{a(m+1)}}{E\, t_{m+1}\, h^2_{m+1}} \tag{5-12}$$

$$\frac{6\,\dfrac{d\, M^{(0)}_{am}}{dx}}{E\, t_m\, h^2_m} + \frac{\dfrac{d\, N^{(1)}_{am}}{dx}}{E\, t_m\, h_m} + \frac{6\,\dfrac{d\, M^{(0)}_{am}}{dx}}{E\, t_m\, h^2_m} =$$

$$-\frac{6\,\dfrac{d\, M^{(0)}_{am+1}}{dx}}{E\, t_{m+1} \cdot h^2_{m+1}} + \frac{\dfrac{d\, N^{(0)}_{a(m+1)}}{dx}}{E\, t_{m+1}\, h_{m+1}} - \frac{6\,\dfrac{d\, M^{1}_{a(m+1)}}{dx}}{E\, t_{m+1}\, h^2_{m+1}} \tag{5-13}$$

$$\frac{1}{t_m h_m}\left(2\,T_{a(m-1)} + 4\,T_{am}\right) + \frac{1}{t_{m+1} h_{m+1}}\left(4\,T_{am} + 2\,T_{a(m+1)}\right) =$$

$$\frac{3\,S_{am}\,(L_a + 2x)}{t_m\,h_m^2} + \frac{3\,S_{a(m+1)}\,(L_a + 2x)}{t_{m+1}\,h_{m+1}^2} \qquad (5\text{-}14)$$

$$T_{am} = T^1_{am}\;(L_a + 2x)$$
$$T_{a(m+1)} = T^1_{a(m+1)}\,(L_a + 2x) \qquad (5\text{-}15)$$

$$\frac{1}{t_m h_m}\left(T^1_{a(m-1)}\right) + 2\left(\frac{1}{t_m h_m} + \frac{1}{t_{m+1} h_{m+1}}\right) T^1_{am} + \frac{1}{t_{m+1} h_{m+1}}\,T^1_{am+1}$$

$$= \frac{3\,S_{am}}{t_m\,h_m^2} + \frac{3\,S_{a(m+1)}}{t_{m+1}\,h_{m+1}^2} \qquad (5\text{-}16)$$

The Bending Moment Resultant M_{am}

The reader may refer to equations (4-20) to (4-23) for details.

$$M_{am} = M^0_{am} + M^{(1)}_{am} \qquad (5\text{-}17)$$

$$\frac{d\,M^{(1)}_{am}}{dx} = -\frac{h_m}{2}\left(T_{a(m-1)} + T_{am}\right) \qquad (5\text{-}18)$$

$$\frac{d\,M^{(1)}_{am}}{dx} = -\frac{h_m}{2}\left(T^1_{a(m-1)} + T^1_{am}\right)\;(L_a + 2x) \qquad (5\text{-}19)$$

$$M^{(1)}_{am} = -\frac{h_m}{2}\left(T^1_{a(m-1)} + T^1_{am}\right)(L_a\,x + x^2) + C \qquad (5\text{-}20)$$

The moment $M^{(1)}_{am} = 0$ at $x = 0$ and at $x = -L_a$. Hence $C = 0$

$$M^{(1)}_{am} = -\frac{h_m}{2}\left(T^1_{a(m-1)} + T^1_{am}\right)(L_a\,x + x^2) \qquad (5\text{-}21)$$

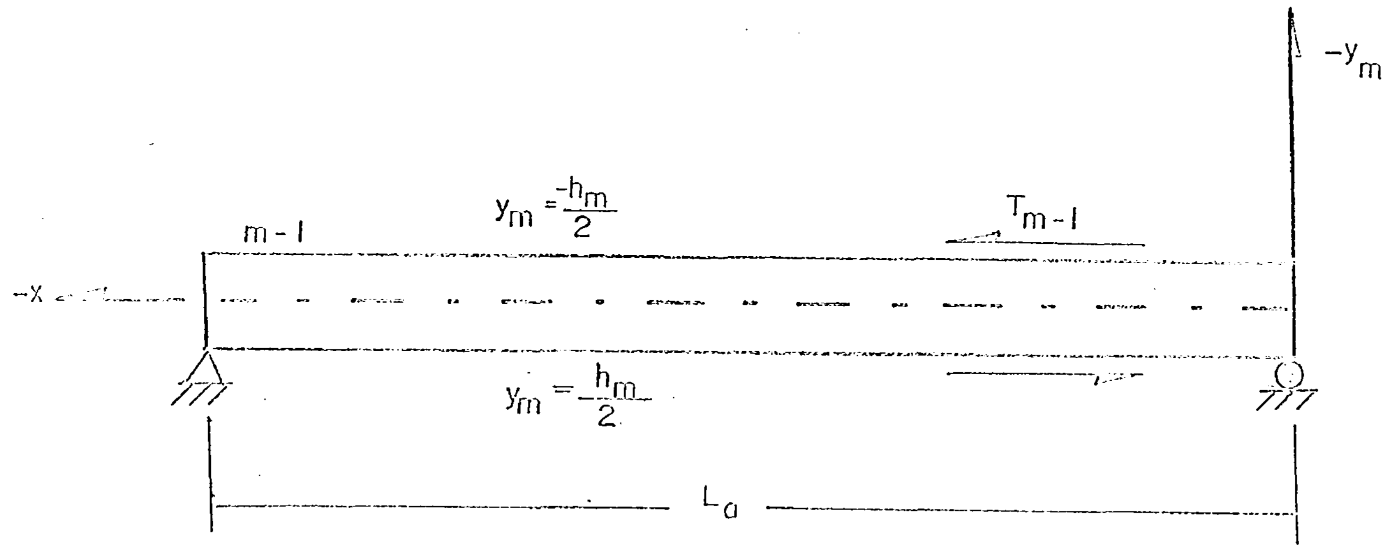


FIG. 11. LONGITUDINAL SHEAR ON PLATE am UNDER UNIFORM LOAD S_{am}

$$M_{am} = -\frac{S_m}{2}\,(L_a x + x^2) - \frac{h_m}{2}\,(T^1_{a(m-1)} + T^1_{am})\,(L_a x \pm x^2) \qquad (5\text{-}22)$$

The Shear Stress Resultant Q_{am}

The reader may refer to equation (4-24) for details.

$$Q_{am} = Q^{(0)}_{am} + Q^{(1)}_{am} \qquad (5\text{-}23)$$

$$Q^{(0)}_{am} = \frac{S_m L_a}{2} - S_m\,(L_a + x) \qquad (5\text{-}24)$$

$$Q^{(1)}_{am} = 0 \qquad (5\text{-}25)$$

$$Q_{am} = -S_m\left(\frac{L_a}{2} + x\right) \qquad (5\text{-}26)$$

The Normal Stress Resultant N_{am}

The reader may refer to equations (4-25) to (4-26) for explanations.

$$N_{am} = N^{(0)}_{am} + N^{(1)}_{am} \qquad (5\text{-}27)$$

$$N^{(0)}_{am} = 0 \qquad (5\text{-}28)$$

$$\frac{d\,N^{(1)}_{am}}{dx} = (T^1_{a(m-1)} - T^1_{am})\,(L_a + 2x) \qquad (5\text{-}29)$$

$$N^{(1)}_{am} = (T^1_{a(m-1)} - T^1_{am})\,(L_a x + x^2) + C \qquad (5\text{-}30)$$

The stress $N^{(1)}_{am} = 0$ at $x = 0$ and at $x = -L_a$. Therefore, $C = 0$.

$$N_{am} = (T^1_{a(m-1)} - T^1_{am})\,(L_a x + x^2) \qquad (5\text{-}31)$$

The Stress Resultant N_{ax}

The reader may refer to equations (4-27) to (4-28) for explanations.

$$\sigma_{axm} = \frac{(T^1_{a(m-1)} - T^1_{am})(L_a x + x^2)}{t_m h_m} + \frac{12\, y_m}{t_m h_m^3}\left[\frac{S_{am}}{2}(L_a x + x^2)\right.$$

$$\left. - \frac{h_m}{2}(T^1_{a(m-1)} + T^1_{m})(L_a x + x^2)\right] \qquad (5-32)$$

$$N_{axm} = t_m \; \sigma_{axm} \qquad (5-33)$$

$$N_{axm} = \frac{L_a x + x^2}{h_m}\left[T^1_{a(m-1)} - T^1_{am} + \frac{6\, S_{am}\, y_m}{h_m^2} \quad \frac{6\, y_m}{h_m}(T^1_{a(m-1)} + T^1_{am})\right] \qquad (5-34)$$

<u>Shear Stress Resultant N_{axym}</u>

The reader may refer to equations (4-29) to (4-34) for explanations.

$$N_{xym} = N^{(0)}_{axym} + N^{(1)}_{axym} \qquad (5-35)$$

$$N^{(0)}_{axym} = \tau^{(0)}_{am}\, t_m = \frac{\dfrac{d\, M^{(0)}_{am}}{dx}}{I_m}\, Q \qquad (5-36)$$

$$Q = \frac{t_m}{8}(h_m^2 - 4\, y_m^2) \qquad (5-37)$$

$$N^{(0)}_{axym} = \frac{3}{4}\;\frac{S_m}{h_m^3}(L_a + 2\,x)\,(h_m^2 - 4\, y_m^2) \qquad (5-38)$$

$$N^{(1)}_{axym} = \tau^{(1)}_{am} t_m\, dx = T^1_{am}(L_a + 2\,x)\, dx + \int_{y_m}^{\frac{h_m}{2}} d\, N_{axm}\, dy \qquad (5-39)$$

$$N^{(1)}_{axm} = \frac{N^{(1)}_{am}}{h_m} + \frac{12\, M^{(1)}_{am}\; y_m}{h_m^3} \qquad (5-40)$$

$$dN^{(1)}_{axm} = \frac{L_a + 2x}{h_m}(T^1_{a(m-1)} - T^1_{am})\, dx - \frac{6(L_a + 2\,x)}{h_m^2}(T^1_{a(m-1)} - T^1_{am})\, y\, dx$$

$$(5-41)$$

$$N^{(1)}_{axym} = \frac{T^1_{am}(L_a + 2x)}{4 h_m^2} (2 y_m + h_m)(6y_m - h_m)$$

$$+ \frac{T^1_{a(m-1)}(L_a + 2x)}{4^2_{hm}} (2 y_m - h_m)(6 y_m + h_m) \qquad (5\text{-}42)$$

$$N_{axym} = \frac{3}{4}\frac{S_{am}}{h_m^3}(L_a + 2x)(h_m^2 - 4 y_m) +$$

$$\frac{T^1_{am}(L_a + 2x)}{4 h_m^2}(2 y_m + h_m)(6 y_m - h_m)$$

$$+ \frac{T^1_{a(m-1)}(L_a + 2x)}{4 h_m^2}(2 y_m \;\; h_m)(6 y_m + h_m) \qquad (5\text{-}43)$$

Rotation of Plate BC

The reader may refer to equations (4-35) to (4-41) for explanations.

$$\frac{d^2 v_{am}}{dx^2} = \frac{-12}{E t_m h_m{}^3} M_{am} \qquad (5\text{-}44)$$

$$\frac{d^2 v_{am}}{dx^2} = \frac{12}{E t_m h_m^3}\left[\frac{-S_{am}}{2}(L_a x + x^2) - \frac{h_m}{2}(T^1_{a(m-1)} + T^1_{am})(L_a x + x^2)\right] \qquad (5\text{-}45)$$

$$\frac{d v_{am}}{dx} = \Theta_{axm} = \frac{12}{E t_m h_m}\left[\frac{-S_{am}}{2}\left(\frac{L_a x^2}{2} + \frac{x^3}{3}\right) - \frac{h_m}{2}(T^1_{a(m-1)} + T^1_{am})\left(\frac{L_a x^2}{2} + \frac{x^3}{3}\right)\right] + C$$

$$(5\text{-}46)$$

at $x = \frac{L}{2}$ $\quad \Theta = 0$

$$C = \frac{1}{2 E t_m h_m^3}\left[S_m L_a^3 + h_m (T'_{a(m-1)} + T'_{am}) L_a^3\right] \qquad (5\text{-}47)$$

$$\theta_{axm} = \frac{12}{E t_m h_m^3}\left[-\frac{S_{am}}{2}\left(\frac{L_a x^2}{2}+\frac{x^3}{3}\right)-\frac{h_m}{2}\left(T^1_{a(m-1)}+T^1_{am}\right)\left(\frac{L_a x^2}{2}+\frac{x^3}{3}\right)\right]$$

$$+\frac{1}{2 E t_m h_m}\left[S_m L_a^3 + h_m\left(T^1_{a(m-1)}+T^1_{am}\right) L_a^3\right] \qquad (5\text{-}48)$$

$$\theta_{a(x=o)m} = \frac{+1}{2 E t_m h_m}\left[S_m L_a^3 + h_m\left(T^1_{a(m-1)}+T^1_{am}\right) L_a^3\right] \qquad (5\text{-}49)$$

Deflection of Plate BC

The reader may refer to equations (4-42) to (4-43) for explanations.

$$v_{axm} = \frac{1}{2 E t_m h_m^3}\left[-S_{am}\left(2 L_a x^3 + x^4\right) - h_m\left(T^1_{a(m-1)}+T^1_{am}\right)\right.$$

$$\left.\left(2L_a x^3 + x^4\right)\right] + Cx + D \qquad (5\text{-}50)$$

At x=o and at x =-L_a v_{axm} = O. Hence D = O and C has the same value as shown in equations (3-10).

$$v_{axm} = -\frac{1}{2 E t_m h_m}\left[\left(x^4 + 2 L_a x^3 - L_a^3 x\right)\left(S_{am} + h_m\left\langle T^1_{a(m-1)} + T^1_{am}\right\rangle\right)\right].$$

(5-51)

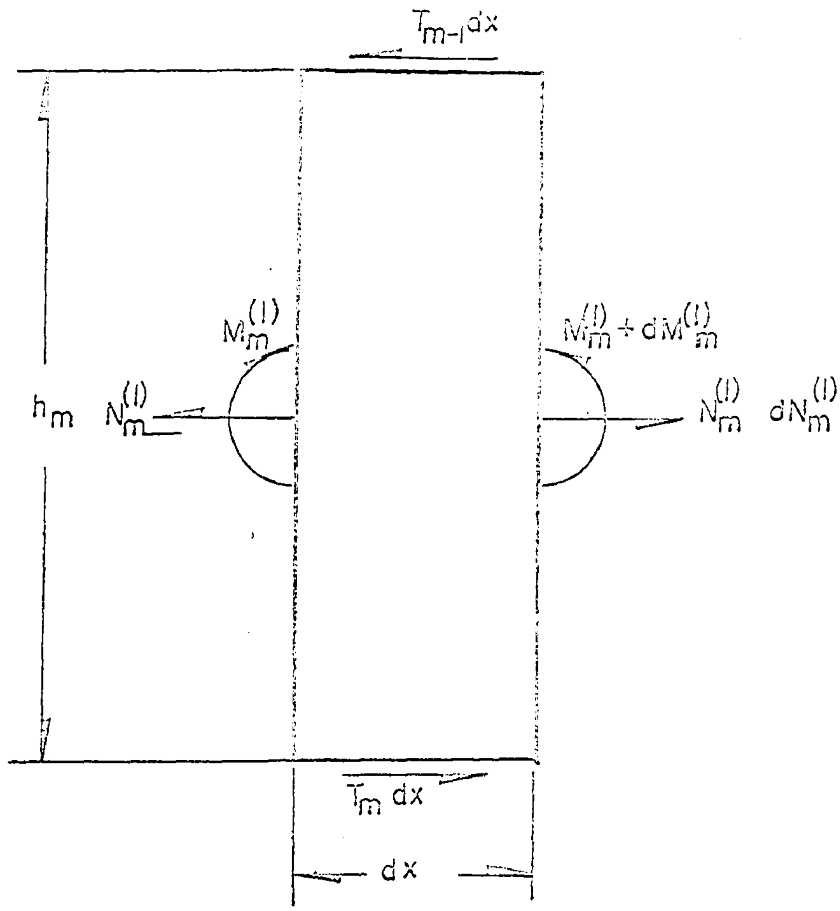


FIG. 12. STRIP ELELEMENT OF R am UNDER UNIFORM LOAD S_{am}

CHAPTER VI

ANALYSIS OF PLATE bm SUBJECTED TO A UNIT MOMENT AT B

Bending Moment Due to M^1_{om}

A unit moment M^1_{om} is applied at the Bend of plate m, span BC, as shown in Fig. 13 (e). The bending moment in the plate

$$M^{(0)r}_{dm} = -1 + \frac{x}{L_b} \tag{6-1}$$

Where the subscript d in $M^{(0)r}_{dm}$ is used to distinguish this moment from $M^{(0)r}_{am}$, $M^{(0)r}_{bm}$, and $M^{(0)r}_{cm}$ which will be introduced later. The super script r indicates the plate to which the unit moment is applied.

Shearing Forces Due to M^1_{om}

$$Q^{(0)(r)}_{dm} = \frac{1}{L} \tag{6-2}$$

Since, as mentioned in Chapter IV, h_m is much smaller than L_b, the bending stress σ_{dxm} and the shear stress $\tau^{(0)}_{dm}$ may be found from elementary beam theory, and so may their products with t_m.

$$N^{(0)r}_{dxym} = \tau^{(0)r}_{dm}\, t_m = \frac{d M^{(0)r}_{dm}}{dx} \frac{A\bar{y}}{I_m}$$

$$N^{(0)r}_{dxym} = \frac{Q^{(0)}_{dm}\left(\frac{h_m}{2} - y_m\right)\left[y_m + \frac{1}{2}\left(\frac{h_m}{2} - y_m\right)\right] t_m}{\frac{1}{12}\; t_m\, h^3_m} \tag{6-3}$$

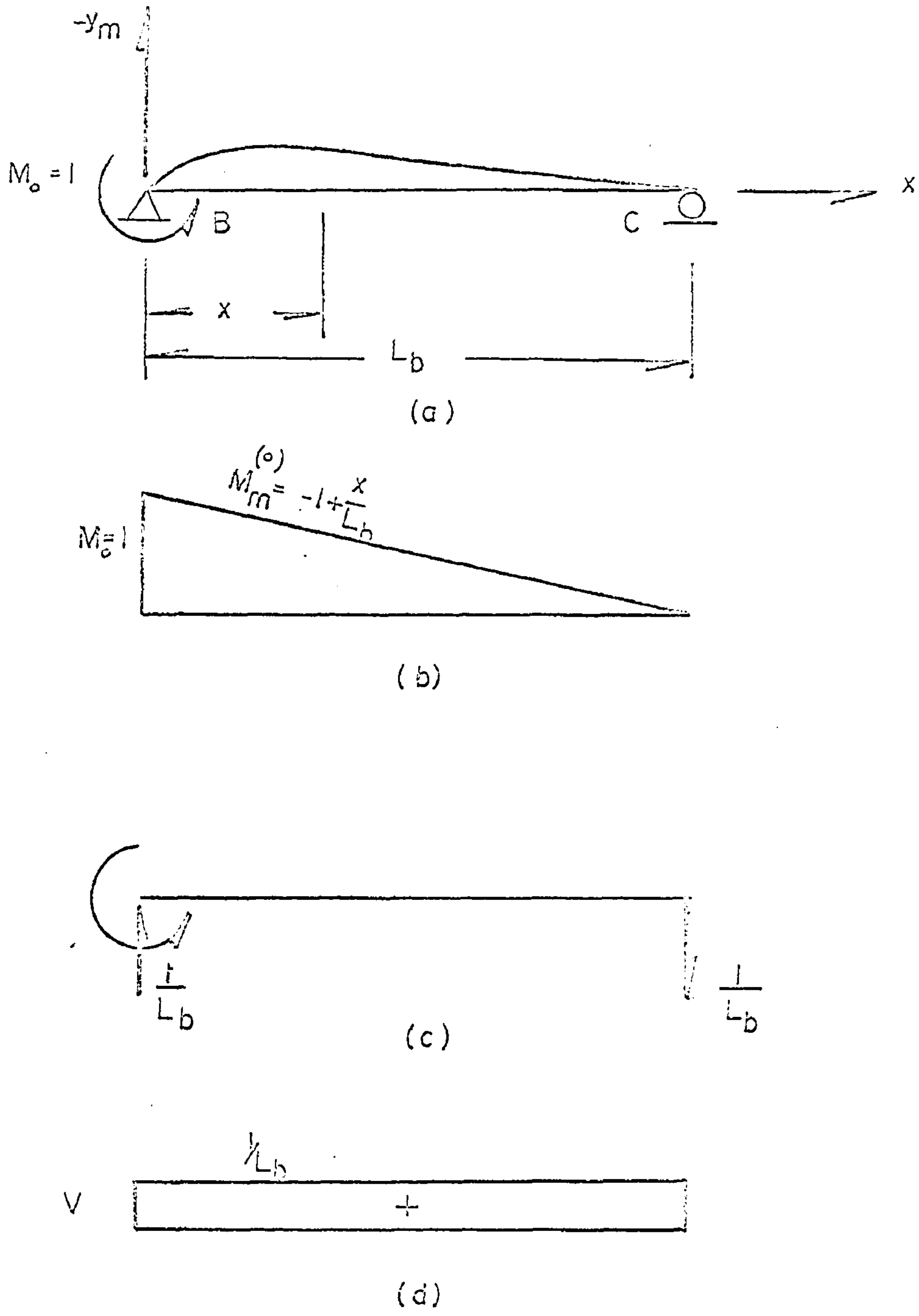


FIG. 13. PLATE bm WITH UNIT MOMENT AT B

Simplifying (6-3)

$$N^{(0)}_{dxym} = \frac{6\,M^{(0)}_{dm}}{h^3_m}\left(\frac{1}{4} - \frac{y^{(2)}_m}{h^2_m}\right) \qquad (6\text{-}4)$$

Also the stress

$$N^{(0)r}_{dxm} = \frac{12\,M^{(0)r}_{dm}\,y_m}{h^3_m} \qquad (6\text{-}5)$$

Longitudinal Shear Stress Equations

At the lower edge of the plate ($y_m = +\frac{h_m}{2}$) the normal force $N^{(0)}_{dxm}$ produces the strain

$$\epsilon^{(0)}_{dxm} = \frac{N^{(o)}_{dxm}}{E\,t_m} = \frac{6\,M^{(0)}_{dm}}{E\,t_m h^2_m} \qquad (6\text{-}6)$$

and the strain at the upper edge ($y_{m+1} = -\frac{h_{m+1}}{2}$) of the adjacent strip is

$$\epsilon^{(0)}_{dx(m+1)} = -\frac{6\,M^{(0)}_{d(m+1)}}{E\,t_{m+1}h^2_{m+1}} \qquad (6\text{-}7)$$

Since the strips are connected to each other, these strains ought to be equal. By making the strains equal the two strips will excert forces upon each other. These additional forces are shearing forces T^r_{dm} acting on the plane of both strips. These forces are shown in Fig. 14 acting in a positive direction in accordance with the sign convention for N_{xy} (Fig. 6 b).

From Fig. 14 it is seen that the moment caused by the longitudinal shears is balanced by the moment caused by the vertical shears. Hence the moment $M^{(1)r}_{dm} = 0$. Since the unit moment is applied only at one plate, say

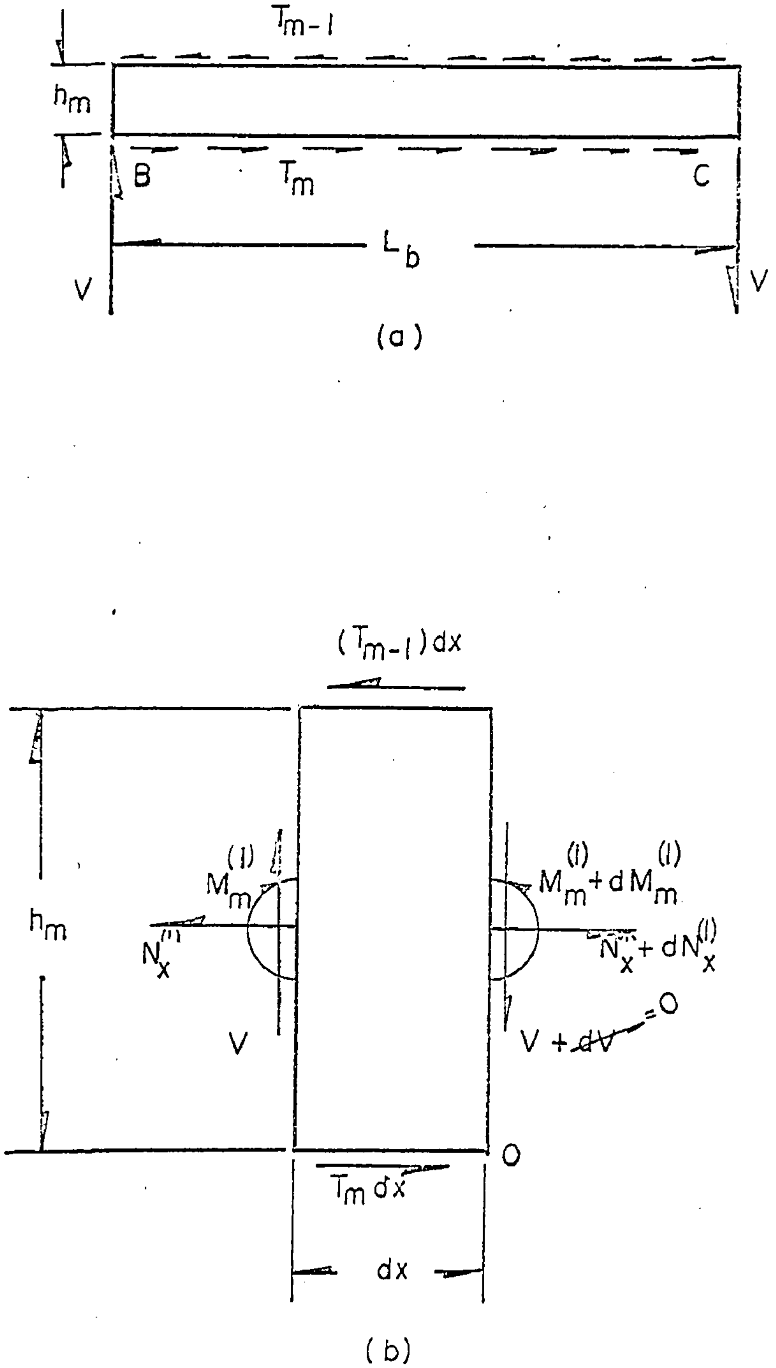


FIG. 14. LONGITUDINAL SHEAR ON ℄ am WITH UNIT MOMENT AT END B

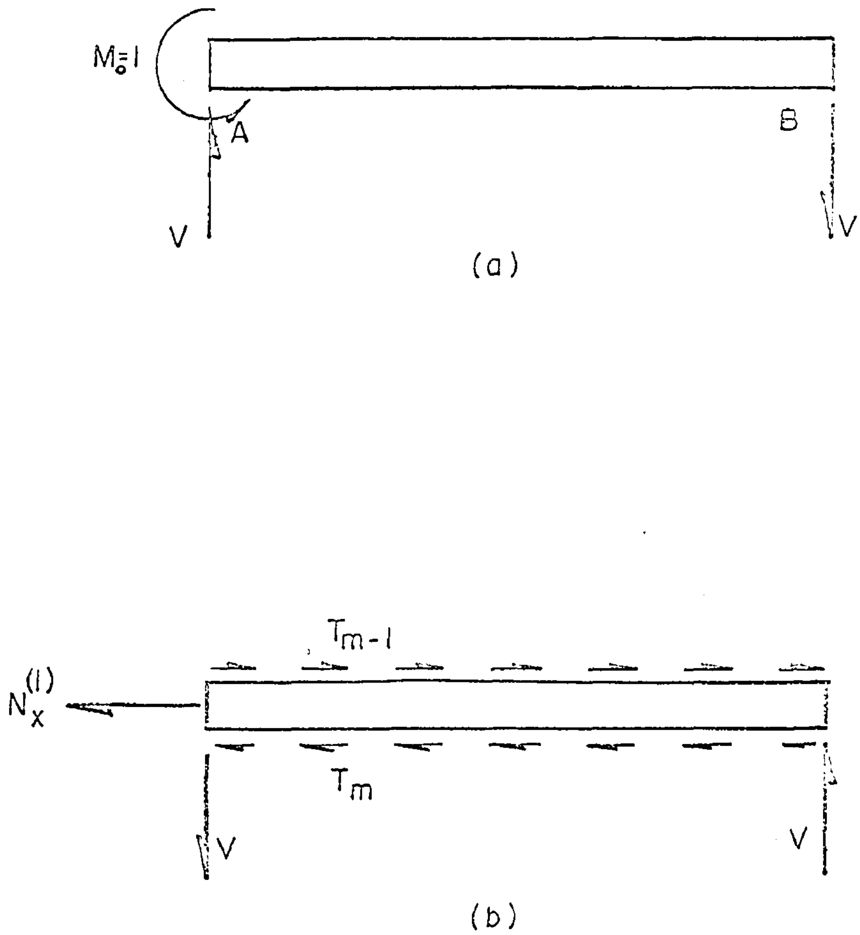


FIG. 15. LONGITUDINAL SHEAR ON ⅊ am WITH UNIT MOMENT AT END B

at $r = m$, then $M^{(0)r}_{d(m+1)} = M^{(0)r}_{d(m-1)} = 0$. From Fig 14

$$\frac{d N^{(1)}_{dm}}{dx} = T_{d(m-1)} - T_{dm} \tag{6-8}$$

$$V dx = +T_{d(m-1)} \frac{h_m}{2} dx + d M^{(1)}_{cm} + T_{dm} \frac{h_m}{2} dx$$

$$d M^{(1)}_{dm} = V_{dx} - \frac{h_m}{2} (T_{d(m-1)} + T_{dm}) dx$$

$$d M^{(1)}_{dm} = \frac{h_m}{2} (T_{dm} + T_{d(m-1)}) dx - \frac{h_m}{2} (T_{d(m-1)} + T_{dm}) dx$$

$$d M^{1}_{dm} = 0 \tag{6-9}$$

The shearing forces T_{dm} must be known in terms of x before equation (6-8) can be integrated.

The shearing forces T_{dm} produce the force $N^{(1)}_{dm}$ from which the normal force

$$N^{(1)}_{dxm} = \frac{N^{(1)}_{dm}}{h_m} \tag{6-10}$$

and the edge strain

$$\epsilon^{(1)}_{dxm} = \frac{N^{(1)}_{dm}}{E\, t_m h_m} \tag{6-11}$$

The strains between the two strips m and m + 1 are made equal by the following relation.

$$\epsilon^{(0)}_{dxm} + \epsilon^{(1)}_{dxm} = \epsilon^{(0)}_{dxm+1} + \epsilon^{(1)}_{dxm+1} \tag{6-12}$$

Where m goes from 1 to K. After having substituted expressions for the ϵ's the following set of equations is obtained.

$$
\left.
\begin{aligned}
&\frac{M^{(1)r}_{d(m-2)}}{E\, t_{(m-2)}\, h_{m-2}} = \frac{N^{(1)r}_{d(m-1)}}{E\, t_{m-1} h_{m-1}} \\[2ex]
&\frac{N^{(1)r}_{d(m-1)}}{E\, t_{m-1} h_{m-1}} = - \frac{6\, M^{(0)r}_{dm}}{E\, t_m\, h_m^2} + \frac{N^{(1)r}_{dm}}{E\, t_m\, h_m} \\[2ex]
&\frac{6\, M^{(0)r}_{dm}}{E\, t_m h_m^2} + \frac{N^{(1)r}_{dm}}{E\, t_m h_m} = \frac{N^{(1)r}_{d(m+1)}}{E\, t_{(m+1)} h_{(m+1)}} \\[2ex]
&\frac{N^{(1)r}_{d(m+1)}}{E\, t_{(m+1)} h_{m+1}} = \frac{N^{(0)r}_{d(m+2)}}{E\, t_{(m+2)} h_{m+2}}
\end{aligned}
\right\} \quad (6\text{-}13)
$$

A set of equations (6-13) must be written for every plate from K = 1 to K - 1 where a unit moment may be applied.

Equations (6-13) are differentiated in respect to x.

$$
\left.
\begin{aligned}
&\frac{\dfrac{d\, N^{(1)}_{d(m-2)}}{dx}}{E\, t_{m-2} h_{m-2}} = \frac{\dfrac{d\, N^{(1)}_{d(m-1)}}{dx}}{E\, t_{m-1} h_{m-1}} \\[2ex]
&\frac{\dfrac{d\, N^{(1)}_{d(m-1)}}{dx}}{E\, t_{m-1}\, h_{m-1}} = - \frac{6\,\dfrac{d\, M^{(0)}_{dm}}{dx}}{E\, t_m\, h_m^2} + \frac{\dfrac{d\, N^{(1)}_{dm}}{dx}}{E\, t_m\, h_m} \\[2ex]
&\frac{\dfrac{6\, d\, M^{0}_{dm}}{dx}}{E\, t_m\, h_m^2} + \frac{\dfrac{d\, N^{(1)}_{dm}}{dx}}{E\, t_m\, h_m} = \frac{\dfrac{d\, N^{(1)}_{d\,(m+1)}}{dx}}{E\, t_{m+1}\, h_{m+1}}
\end{aligned}
\right\} \quad (6\text{-}14)
$$

$$\left.\begin{array}{l} \dfrac{\dfrac{d N^{(1)}_{d(m+1)}}{dx}}{E\, t_{(m+1)}\, h_{(m+1)}} = \dfrac{\dfrac{d N^{(1)}_{d(m+2)}}{dx}}{E\, t_{(m+2)}\, h_{(m+2)}} \\ \text{-------------------------} \end{array}\right\} \qquad (6\text{-}14)$$

From equations (6-1), (6-8), (6-9) and (6-14)

$$\left.\begin{array}{l} \text{-------------------------} \\ \dfrac{T_{d(m-3)} - T_{d(m-2)}}{E\, t_{m-2}\, h_{m-2}} = \dfrac{T_{d(m-2)} - T_{d(m-1)}}{E\, t_{m-1}\, h_{m-1}} \\ \\ \dfrac{T_{d(m-2)} - T_{d(m-1)}}{E\, t_{(m-1)}\, h_{(m-1)}} = \dfrac{-6/L_b}{E\, t_m h_m^2} + \dfrac{T_{d(m-1)} - T_{d(m)}}{E\, t_m\, h_m} \\ \\ \dfrac{6/L_b}{E\, t_m h_m^2} + \dfrac{T_{d(m-1)} - T_{dm}}{E\, t_m\, h_m} = \dfrac{T_{d(m)} - T_{d(m+1)}}{E\, t_{m+1}\, h_{m+1}} \\ \\ \dfrac{T_{d(m)} - T_{d(m+1)}}{E\, t_{m+1}\, h_{m+1}} = \dfrac{T_{d(m+1)} - T_{d(m+2)}}{E\, t_{m+2}\, h_{m+2}} \\ \text{-------------------------} \end{array}\right\} \qquad (6\text{-}15)$$

Equations (6-15) give the distribution of shears T_{dm} throughout the structure due to a unit moment applied at the end B of plate m in span BC. Similar sets of equations result when the unit moment is applied at different plates. Hence K-1 sets of equations (6-15) may be written for a particular folded plate structure.

It may be seen from equations (6-15) that the absolute term $\dfrac{6/L_b}{E\, t_m\, h_m^2}$ is independent of x. Hence the shear stress T_{dm} are also independent of x and are constant quantities.

<u>The Bending Moment Resultant M^x_{dm}</u>

The bending moment resultant of strips m-2, m-1, m, ------ in span BC due to a unit moment applied at plate r=m may be expressed as

$$\left.\begin{array}{l} \text{- - - - - - - - - - - - - - - -} \\ M^{r}_{d(m-1)} = M^{(0)r}_{d(m-1)} + M^{(1)r}_{d(m-1)} \\ M^{r}_{dm} = M^{(0)r}_{dm} + M^{(1)r}_{dm} \\ M^{r}_{d(m+1)} = M^{(0)r}_{d(m+1)} + M^{(1)r}_{d(m+1)} \\ \text{- - - - - - - - - - - - - - - -} \end{array}\right\} \tag{6-16}$$

Since the terms on the right hand side of equation (6-16) are all equal to zero except for $M^{(0)}_{dm}$ then equations (6-16) may be reduced to

$$M^{r}_{dm} = M^{(0)r}_{dm} \tag{6-17}$$

where r=m. Hence

$$M^{r}_{dm} = -1 + \frac{x}{L_b} \tag{6-18}$$

The Shear Resultant Q_{dm}

The total shears stress resultant

$$Q_{dm} = Q^{(0)}_{dm} + Q^{(1)}_{dm} \tag{6-19}$$

But $Q^{(1)}_{dm} = 0$ since the vertical shears cancel each other. Hence, substituting equation (6-2) into equation (6-19)

$$Q^{r}_{dm} = \frac{1}{L_b} \tag{6-20}$$

where r = m.

The Normal Stress Resultant N_{dm}

The total normal stress resultant

$$N_{dm} = N^{(0)}_{dm} + N^{(1)}_{dm} \tag{6-21}$$

But $N^{(0)}_{dm} = 0$ since the applied unit moment at B does not produce any net force in the x-direction at a given cross section. Equation (6-21) is differentiated.

$$\frac{d\,N_{dm}}{dx} = \frac{d\,N^{(1)}_{dm}}{dx} \tag{6-22}$$

Equation (6-8) is substituted into equation (6-22) and the result is integrated.

$$\frac{d\,N_{dm}}{dx} = T_{d(m-1)} - T_{dm} \tag{6-23}$$

$$N_{dm} = (T_{d(m-1)} - T_{dm})\,x + C$$

At $x = L_b$ $\quad N_{dm} = 0$ (Fig. 15).

$$C = -\,(T_{d(m-1)} - T_{dm})\,L_b$$

$$N_{dm} = (T_{d(m-1)} - T_{dm})\,(x - L_b) \tag{6-24}$$

The set of equations for the normal stress resultant N^r_{dm} due to a unit moment applied at plate r = m is the following.

$$\left.\begin{array}{l}
-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\\
N^r_{d(m-2)} = (T^r_{d(m-3)} - T_{d(m-2)})\,(x - L_b)\\
N^r_{d(m-1)} = (T^r_{d(m-2)} - T^r_{d(m-1)})\,(x - L_b)\\
N^r_{dm} = (T^r_{d(m-1)} - T^r_{dm})\,(x - L_b)\\
N^r_{d(m+1)} = (T^r_{dm} - T^r_{d(m+1)})\,(x - L_b)\\
-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-\,-
\end{array}\right\} \tag{6-25}$$

A set of such equations may be written for the unit moment applied at each different plate from plate K = 1 to K - 1.

The Stress Resultant N_{dx}

From elementary theory

$$\sigma_x = \frac{P}{A} \pm \frac{My}{I}$$

Then

$$\sigma^r_{dxm} = \frac{N_{dm}}{t_m h_m} + \frac{y_m M_{dm}}{\frac{1}{12} t_m h_m^3} \tag{6-26}$$

$$\sigma^r_{dxm} = \frac{(T_{d(m-1)} - T_{dm})(x - L_b)}{t_m \; h_m} = \frac{12\, y_m}{t_m h_m^3}\left(-1 + \frac{x}{L_b} \right) \tag{6-27}$$

$$N^r_{dxm} = \sigma^r_{d(xm)} \, t_m$$

Hence we may write the set of equations for the stress resultant N^r_{dxm} due to a unit moment applied at plate m in span BC.

$$\left.\begin{aligned}
N^{(r)}_{dx(m-1)} &= \frac{(T^{(r)}_{d(m-2)} - T^{(r)}_{d(m-1)})(x - L_b)}{h_m} \\
N^{(r)}_{dxm} &= \frac{(T^{(r)}_{d(m-1)} - T^{(r)}_{dm})(x - L_b)}{h_m} + \frac{12 y_m}{t_m h_m^3}\left(-1 + \frac{x}{L_b} \right) \\
N^{(r)}_{dx(m+1)} &= \frac{1}{h_m}\,(T^{(r)}_{dm} - T^{(r)}_{d(m+1)})\,(x - L_b)
\end{aligned}\right\} \tag{6-28}$$

A total number of K-1 of such sets may be written as the unit moment is transferred from plate to plate.

Shear Stress Resultant N^{r}_{bxym}

$$N^{(r)}_{dxym} = N^{(0)r}_{dxym} + N^{(1)r}_{dxym} \tag{6-29}$$

$$N^{0(r)}_{dxym} = \tau^{(0)r}_{dm}\, t_m = \frac{\dfrac{d M^{(0)r}_{dm}\; Q}{dx}}{I_m} \tag{6-30}$$

$$I_m = \frac{1}{12} t_m h_m^3 \tag{6-31}$$

$$Q = A\bar{y} = \frac{t_m}{8}\left(h_m^2 - 4y^2{}_m\right) \tag{6-32}$$

$$\frac{d M^{(0)r}_{dm}}{dx} = \frac{1}{L_b} \tag{6-33}$$

Equations (6-31) to (6-33) are substituted into equation (6-30)

$$N^{(0)r}_{dxym} = \frac{3}{2}\,\frac{\left(h_m^2 - 4y^2{}_m\right)}{L_b\, h_m^3} \tag{6-34}$$

Fig. 16 is analyzed next to determine $N^{(1)}_{dxym}$.

$$\tau^{(1)r}_{dm}\, t_m\, d_x = \tau_{dm}\, d\,x + \int_{y_m}^{\frac{h_m}{2}} d\,N^{(1)r}_{dxm}\, dy \tag{6-35}$$

But

$$N^{(1)r}_{dxm} = \frac{N^{(1)r}_{dm}}{h_m} + \frac{12\, M^{(1)r}_{dm}\, y_m}{h_m^3} \tag{6-36}$$

Equations (6-8) and (6-9) are substituted in equation (6-36).

$$N^{(1)r}_{dxm} = \frac{1}{h_m}\left(T^{r}_{d(m-1)} - T^{r}_{dm}\right) \tag{6-37}$$

Equation (6-37) is differentiated in respect to x and then substituted in equation (6-35). The term dx is common and is therefore dropped.

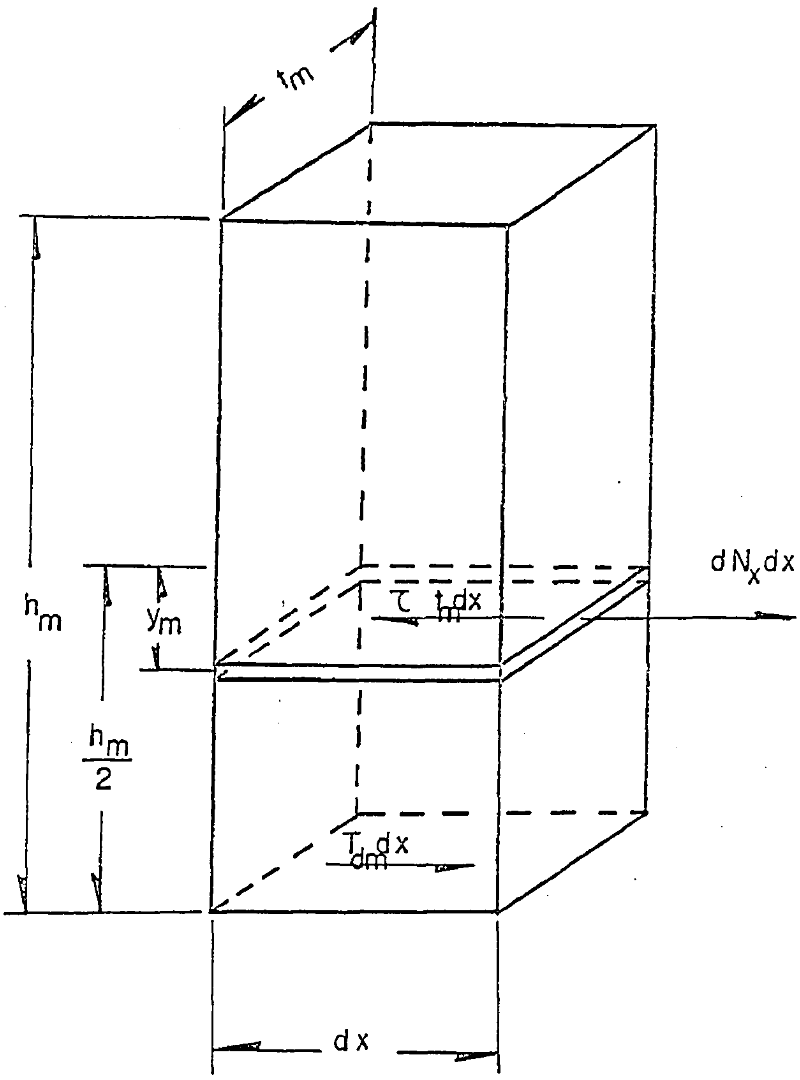


FIG. 16. VOLUME ELEMENT OF R bm
WITH UNIT MOMENT AT END B

$$\tau^{(1)r}_{dm}\, t_m = T_{dm} + \int_{y_m}^{\frac{h_m}{2}} \frac{1}{h_m}\left(T^{(r)}_{d(m-1)} - T^{(r)}_{dm}\right) dy \qquad (6\text{-}38)$$

$$N^{(1)r}_{dxym} = \tau^{(1)r}_{dm}\, t_m = T^{(r)}_{dm} - \frac{1}{h_m}\left(T^{(r)}_{d(m-1)} - T^{(r)}_{dm}\right)\left(y_m - \frac{h_m}{2}\right) \qquad (6\text{-}39)$$

Equations (6-34) and (6-39) are substituted in equation (6-29) to give the following set of equations

$$\left.\begin{aligned}
N^{(r)}_{dxy(m-1)} &= T^{(r)}_{d(m-1)} - \frac{1}{h_{m-1}}\left(T^{(r)}_{d(m-2)} - T^{(r)}_{d(m-1)}\right)\left(y_{m-1} - \frac{h_{m-1}}{2}\right)\\
N^{(r)}_{dxym} &= \frac{3}{2}\,\frac{\left(h_m^2 - 4y_m^2\right)}{L_b\, h_m^3} + T^{(r)}_{dm} - \frac{1}{h_m}\left(T^{(r)}_{d(m-1)} - T^{(r)}_{dm}\right)\left(y_m - \frac{h_m}{2}\right)\\
N_{dxy(m+1)} &= T^{(r)}_{d(m+1)} - \frac{1}{h_{m+1}}\left(T^{(r)}_{dm} - T^{(r)}_{d(m+1)}\right)\left(y_{m+1} - \frac{h_{m+1}}{2}\right)
\end{aligned}\right\} \quad (6\text{-}40)$$

The set of equations (6-40) determine the distribution of the shear stress resultant $N^{(r)}_{bxym}$ throughout the structure due to a unit moment applied at end B of plate m in span BC. A total number of K-1 sets of such equations may be written as the unit moment is moved from plate K = 1 to plate K - 1.

Rotation and Deflection of Plate bm

The deflection of plate bm is subject to the differential equation

$$\frac{d^2 v^{(r)}_{dm}}{dx^2} = \frac{-12}{E\, t_m h_m^3}\, M_{dm} \qquad (6\text{-}41)$$

Equation (6-18) is substituted into equation (6-41)

$$\left.\frac{d^2 v^{(r)}_{dm}}{dx^2} = \frac{-12}{E\, t_m h_m^3}\left(-1 + \frac{x}{L_b}\right)\right\} \qquad (6\text{-}42)$$

If integrated once in respect to x, equation (6-42) will yield the slope of the elastic curve of plate m in span BC due to a unit moment applied at r = m.

$$\frac{d\,v^{(r)}_{dm}}{dx} = \theta_{dxm} = \frac{12}{E\,t_m h_m^3} \left(-x + \frac{x^2}{2\,L_b} \right) + C \tag{6-43}$$

If equation (6-42) is integrated twice in respect to x, then the equation of the elastic curve is obtained.

$$v^{(r)}_{dm} = \frac{12}{E\,t_m\,h_m^3}\left(-\frac{x^2}{2} + \frac{x^3}{6\,L_b} \right) + Cx + D \tag{6-44}$$

The deflection $v^{(r)}_{dm} = 0$ at $x = 0$ and at $x = L_b$. These conditions are substituted in equation (6-44) to find

$$D = 0 \tag{6-45}$$

$$C = \frac{4\,L_b}{E\,t_m\,h_m^3} \tag{6-46}$$

Equation (6-46) is substituted in equation (6-43) to find the rotation of plate m.

$$\theta^{(r)}_{dxm} = \frac{12}{E\,t_m h_m^3}\left(-x + \frac{x^2}{2\,L_b} \right) + \frac{4\,L_b}{E\,t_m\,h_m^3} \tag{6-47}$$

$$\theta^{(r)}_{d(x=o)m} = \frac{4\,L_b}{E\,t_m\,h_m^3} \tag{6-48}$$

Equations (6-45) and (6-46) are substituted in equation (6-43) to yield the equation of the deflection curve

$$v^{(r)}_{dm} = \frac{12}{E\,t_m\,h_m^3}\left(\frac{x^3}{6\,L_b} - \frac{x^2}{2} + \frac{L_b\,x}{3} \right) \tag{6-49}$$

CHAPTER VII

ANALYSIS OF PLATE a m SUBJECT TO UNIT MOMENT APPLIED AT END B

Plate a m (Fig. 17) is analyzed in the same way as plate bm in Chapter VI. Most of the equations in this chapter are the same as those appearing in Chapter VI. The only differences may be in signs. For this reason explanations have been omitted in this section. The reader may refer back to Chapter VI for more details.

Bending Moment Due to Unit Moment Applied at B.

$$M^{(0)r}_{cm} = -1 - \frac{x}{L_a} \qquad (7\text{-}1)$$

Shearing Forces Due to M^{1}_{cm}

$$Q^{(0)r}_{cm} = -\frac{1}{L_a}$$

$$N^{(0)r}_{cxym} = \tau^{(0)r}_{cm}\, t_m = \frac{d\,M^{(0)r}_{cm}}{dx} \quad \frac{A\bar{y}}{I_m} \qquad (7\text{-}2)$$

$$N^{(0)r}_{cxym} = \frac{Q^{(0)r}_{cm}\left(\frac{h_m}{2} - y_m\right)\left[y_m + \frac{1}{2}\left(\frac{h_m}{2} - y_m\right)\right]\ t_m}{\frac{1}{12}\, t_m\, h^3_m} \qquad (7\text{-}3)$$

Simplifying (7-3)

$$N^{(0)r}_{cxym} = \frac{6\,Q^{(0)}_{cm}}{h^3_m}\left(\frac{1}{4} - \frac{y^2_m}{h^2_m}\right) \qquad (7\text{-}4)$$

$$N^{(0)r}_{cxm} = \frac{12\,M^{(0)r}_{cm}\ y_m}{h^3_m} \qquad (7\text{-}5)$$

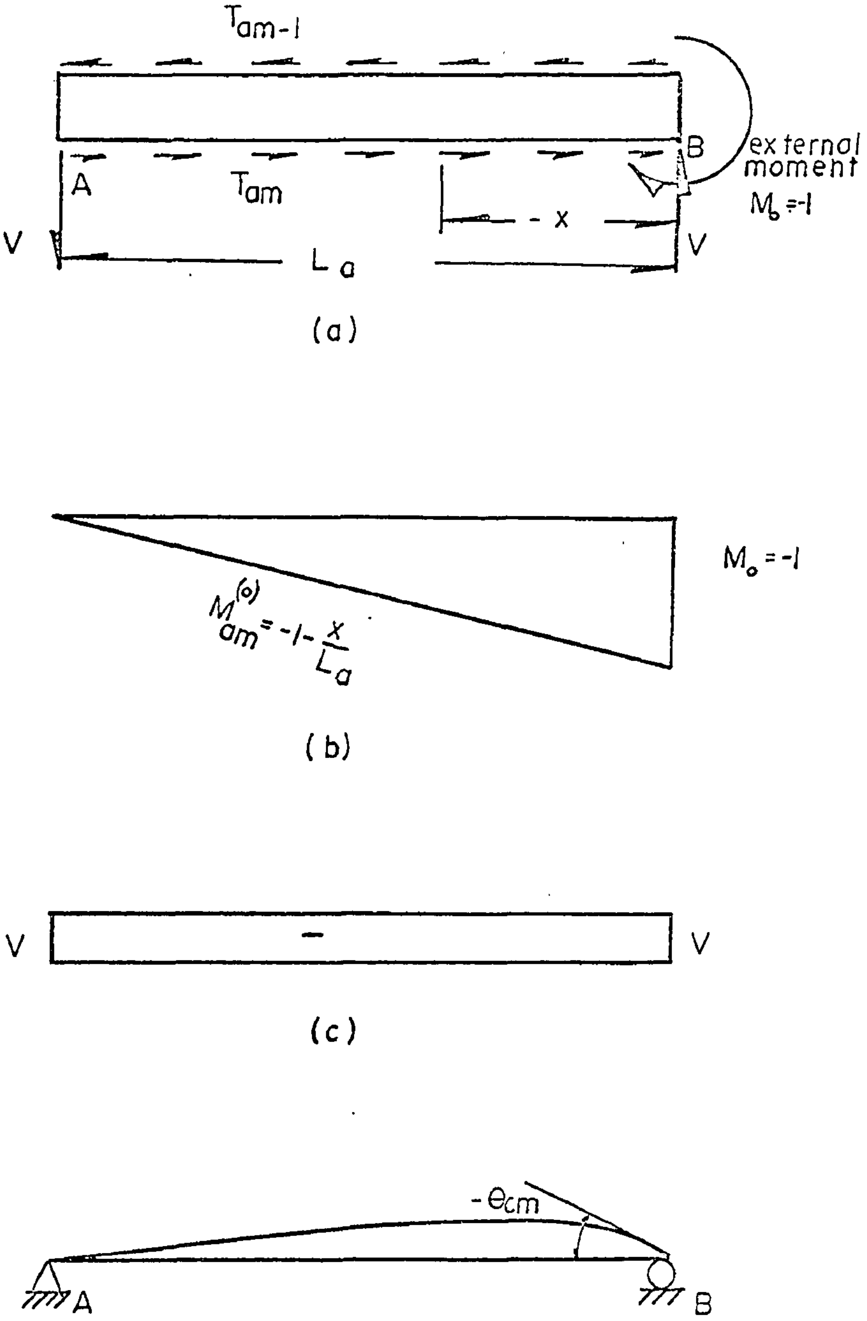


FIG. 17. PLATE am WITH UNIT MOMENT AT B

Longitudinal Shear Stress Equations

at $y_m = -\frac{h_m}{2}$

$$\epsilon^{(0)r}_{cxm} = \frac{N^{(0)r}_{cxm}}{E\, t_m} = \frac{6\, M^{(0)r}_{cm}}{E\, t_m\, h_m^2} \tag{7-6}$$

at $y_{m+1} = -\frac{h_{m+1}}{2}$

$$\epsilon^{(0)r}_{cxm+1} = -\frac{6\, M^{(0)r}_{c(m+1)}}{E\, t_{m+1}\, h^2_{m+1}} \tag{7-7}$$

Since the unit moment is applied only at one plate, say at $r = m$, then

$M^{(0)r}_{c(m+1)} = M^{(0)r}_{c(m-1)} = 0.$

From Fig. 18

$$d\, M^{(1)r}_{cm} = (T_{c(m-1)} + T_{cm})\, \frac{h_m}{2}\, dx - V dx \tag{7-8}$$

$$d\, M^{(1)r}_{cm} = \frac{h_m}{2}\,(T_{c(m-1)} + T_{cm})\, dx - \frac{h_m}{2}(T_{c(m-1)} + T_{cm})$$

$$d\, M^{(1)r}_{cm} = 0$$

$$\frac{d\, N^{(1)r}_{cm}}{dx} = -T_{c(m-1)} + T_{cm} \tag{7-9}$$

$$N^{(1)r}_{cxm} = \frac{N^{(1)r}_{cm}}{h_m} \tag{7-10}$$

$$\epsilon^{(1)r}_{cxm} = \frac{N^{(1)r}_{cm}}{E\, t_m\, h_m} \tag{7-11}$$

The equation of compatibility is

$$\epsilon^{(0)}_{cxm} + \epsilon^{(1)}_{cxm} = \epsilon^{(0)}_{cx(m+1)} + \epsilon^{(1)}_{cx(m+1)} \tag{7-12}$$

from which

$$
\left.
\begin{aligned}
&\frac{N^{(1)r}_{c(m-2)}}{E\, t_{m-2}\, h_{m-2}} = \frac{N^{(1)r}_{c(m-1)}}{E\, t_{m-1} h_{m-1}} \\[2ex]
&\frac{N^{(1)r}_{c(m-1)}}{E\, t_{m-1} h_{m-1}} = -\frac{6\, M^{(0)r}_{cm}}{E\, t_m\, h_m^2} + \frac{N^{(1)r}_{cm}}{E\, t_m\, h_m} \\[2ex]
&\frac{6\, M^{(0)r}_{cm}}{E\, t_m\, h_m^2} + \frac{N^{(1)r}_{m}}{E\, t_m\, h_m} = \frac{N^{(1)r}_{m+1}}{E\, t_{m+1}\, h_{m+1}} \\[2ex]
&\frac{N^{(1)r}_{m+1}}{E\, t_{m+1} h_{m+1}} = \frac{N^{(1)r}_{(m+2)}}{E\, t_{m+2} h_{m+2}}
\end{aligned}
\right\} \quad (7\text{-}13)
$$

The shears T_{cm} are substituted in the differentiated form of equations (7-13).

$$
\left.
\begin{aligned}
&\frac{-T^r_{c(m-3)} + T^r_{c(m-2)}}{E\, t_{m-2}\quad h_{m-2}} = \frac{-T^r_{c(m-2)} + T^r_{c(m-1)}}{E\, t_{m-1}\quad h_{m-1}} \\[2ex]
&\frac{-T^r_{c(m-2)} + T^r_{c(m-1)}}{E\, t_{m-1}\quad h_{m-1}} = \frac{+1/L_a}{E\, t_m\, h_m^2} + \frac{-T_{c(m-1)} + T_{cm}}{E\, t_m\, h_m} \\[2ex]
&\frac{-1/L_a}{E\, t_m\, h_m^2} + \frac{-T_{c(m-1)} + T_{cm}}{E\, t_m\, h_m} = \frac{-T_{cm} + T_{c(m+1)}}{E\, t_{m+1}\quad h_{m+1}} \\[2ex]
&\frac{-T_{cm} + T_{c(m+1)}}{E\, t_{m+1}\quad h_{m+1}} = -\frac{T_{c(m+1)} + T_{c(m-2)}}{E\, t_{m+2}\quad h_{m+2}}
\end{aligned}
\right\} \quad (7\text{-}14)
$$

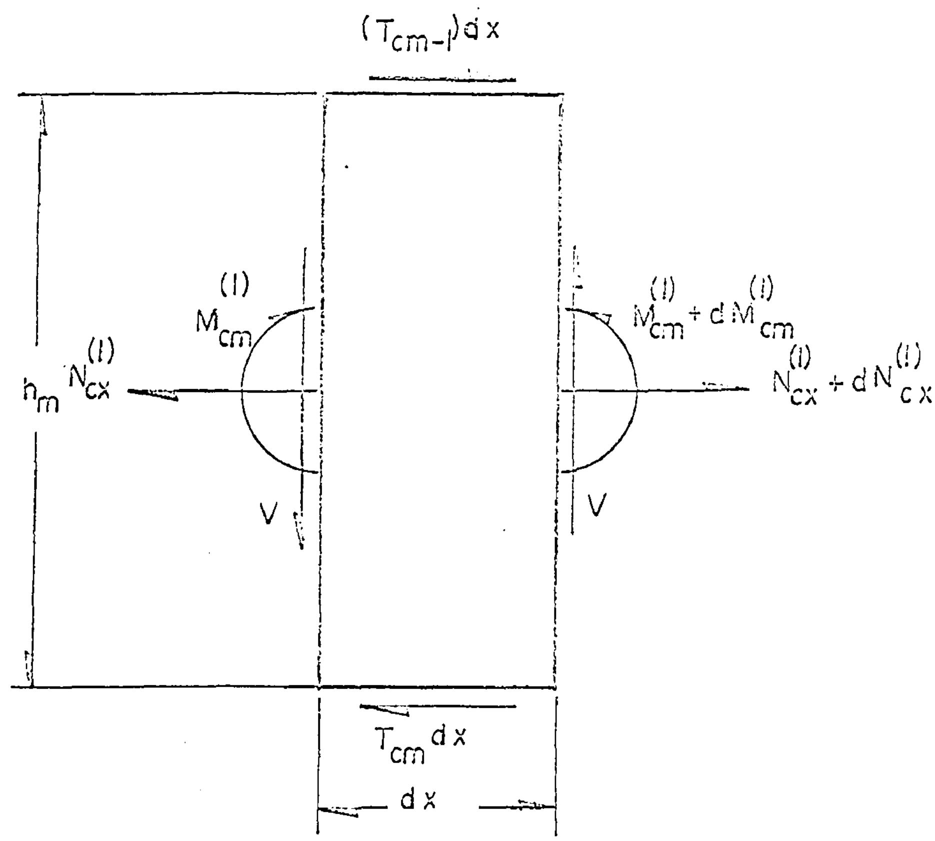


FIG. 16. STRIP ELEMENT OF R am
WITH UNIT MOMENT AT B

The Bending Moment $M^{(r)}_{cm}$

$$\left.\begin{array}{l}
\text{- - - - - - - - - -}\\
M^{r}_{c(m-1)} = M^{(0)r}_{c(m-1)} + M^{(1)r}_{c(m-1)}\\
M^{r}_{cm} = M^{(0)r}_{cm} + M^{(1)r}_{cm}\\
M^{r}_{c(m+1)} = M^{(0)r}_{c(m+1)} + M^{(1)r}_{c(m+1)}\\
\text{- - - - - - - - - -}
\end{array}\right\} \qquad (7\text{-}15)$$

$$M^{(r)}_{cm} = M^{(0)r}_{cm} \qquad (7\text{-}16)$$

$$M^{(r)}_{cm} = -1 - \frac{x}{L_a} \qquad (7\text{-}17)$$

The Shear Resultant Q_{cm}

$$Q^{r}_{cm} = Q^{(0)r}_{cm} + Q^{(1)r}_{cm} \qquad (7\text{-}18)$$

$$Q^{(r)}_{cm} = -\frac{1}{L_a} \qquad (7\text{-}19)$$

The Normal Stress Resultant N_{cm}

$$N^{(r)}_{cm} = N^{(0)r}_{cm} + N^{(1)r}_{cm} \qquad (7\text{-}20)$$

$$\frac{d\,N^{r}_{cm}}{dx} = \frac{d\,N^{(1)r}_{cm}}{dx} \qquad (7\text{-}21)$$

$$\frac{d\,N^{(r)}_{cm}}{dx} = -T^{r}_{c(m-1)} + T^{r}_{cm}$$

$$N^{(r)}_{cm} = \left(-T^{r}_{c(m-1)} + T^{r}_{cm}\right)x + C$$

At $x = -L_a$ $\quad N^r_{cm} = 0$

$$C = (-T^r_{c(m-1)} + T^r_{cm})(-L_a)$$

$$\left.\begin{aligned}
&\text{-------------------------------}\\
&N^r_{c(m-2)} = (-T^r_{c(m-3)} + T^r_{c(m-2)})(x - L_a)\\
&N^r_{c(m-1)} = (-T^r_{c(m-2)} + T^r_{c(m-1)})(x - L_a)\\
&N^r_{cm} = (-T^r_{c(m-1)} + T^r_{cm})(x - L_a)\\
&N^{(r)}_{c(m+1)} = (-T^r_{cm} + T^r_{c(m+1)})(x - L_a)\\
&\text{-------------------------------}
\end{aligned}\right\} \quad (7\text{-}22)$$

The Stress Resultant N_{dx}

$$\sigma^r_{cxm} = \frac{N^r_{cm}}{t_m h_m} + \frac{y_m M^r_{cm}}{\frac{1}{12} t_m h^3_m} \quad (7\text{-}23)$$

$$\sigma^r_{cxm} = \frac{(-T^r_{c(m-1)} + T^r_{cm})(x - L_a)}{t_m h_m} + \frac{12 y_m}{t_m h^3_m}\left(-1 - \frac{x}{L_a}\right) \quad (7\text{-}24)$$

$$N^r_{cxm} = \sigma^r_{cxm} t_m \quad (7\text{-}25)$$

$$\left.\begin{aligned}
&\text{-------------------------------}\\
&N^r_{cx(m-1)} = \frac{(-T^r_{c(m-2)} + T^r_{c(m-1)})(x - L_a)}{h_m}\\
&N^r_{cxm} = \frac{(-T^r_{c(m-1)} + T^r_{cm})(x - L_a)}{h_m} + \frac{12 y_m}{t_m h^3_m}\left(-1 - \frac{x}{L_a}\right)
\end{aligned}\right.$$

$$N^{r}_{cx(m+1)} = \left(\frac{-T^{(r)}_{cm} + T^{r}_{c(m+1)} \; (x - L_a)}{h_m} \right) \qquad (7\text{-}26)$$

<u>Shear Stress Resultant N^{r}_{cxym}</u>

$$N^{r}_{cxym} = N^{(0)r}_{cxym} + N^{(1)r}_{cxym} \qquad (7\text{-}27)$$

$$N^{(0)r}_{cxym} = \tau^{(0)r}_{cm} \, t_m = \frac{\frac{d M^{(0)r}_{c}}{dx} \, Q}{I_m} \qquad (7\text{-}28)$$

$$I_m = \frac{1}{12} \, t_m \, h^3_m$$

$$Q = A\bar{y} = \frac{t_m}{8} \left(h^2_m - 4 y^2_m \right) \qquad (7\text{-}29)$$

$$\frac{d M^{(0)r}_{cm}}{dx} = - \frac{1}{L_a} \qquad (7\text{-}30)$$

$$N^{(0)r}_{cxym} = - \frac{3}{2} \, \frac{\left(h^2_m - 4 y^2_m \right)}{L_a \, h^3_m} \qquad (7\text{-}31)$$

$$\tau^{(1)r}_{cm} \, t_m \, dx = T^{r}_{cm} \, dx + \int_{y_m}^{\frac{h_m}{2}} d N^{(1)r}_{cxm} \, dy \qquad (7\text{-}32)$$

$$N^{(1)r}_{cxm} = \frac{N^{(1)r}_{cm}}{h_m} + \frac{12 \, M^{(1)r}_{cm} \, y_m}{h^3_m}$$

$$N^{(1)r}_{cxm} = \frac{1}{h_m} \left(- T^{r}_{c(m-1)} + T^{r}_{cm} \right) \qquad (7\text{-}33)$$

$$\tau^{(1)r}_{cm} t_m = T^r_{cm} + \int_{y_m}^{\frac{h_m}{2}} \frac{1}{h_m} (- T^r_{c(m-1)} + T^r_{cm}) \, dy \tag{7-34}$$

$$N^{(1)r}_{cxym} = \tau^{(1)r}_{cm} t_m = T^r_{cm} - \frac{1}{h_m} (- T^r_{c(m-1)} + T^r_{cm})(y_m - \frac{h_m}{2}) \tag{7-35}$$

$$\left.\begin{aligned} N^r_{cxy(m-1)} &= T^r_{c(m-1)} - \frac{1}{h_{m-1}} (- T^r_{c(m-2)} + T^r_{c(m-1)})(y_{m-1} - \frac{h_{m-1}}{2}) \\ N^r_{cxym} &= -\frac{3}{2} \frac{(h_m^2 - 4 y_m^2)}{L_a h_m^3} + T^r_{cm} - \frac{1}{h_m} (- T^r_{c(m-1)} + T^r_{cm})(y_m - \frac{h_m}{2}) \\ N^r_{cxy(m+1)} &= T^r_{c(m+1)} - \frac{1}{h_{m+1}} (- T^r_{cm} + T^r_{c(m+1)})(y_{m+1} - \frac{h_{m+1}}{2}) \end{aligned}\right\} \tag{7-36}$$

<u>Rotation and Deflection of Plate am</u>

$$\frac{d^2 v_{cm}}{dx^2} = -\frac{12}{E t_m h_m^3} (- 1 - \frac{x}{L_a}) \tag{7-37}$$

$$\frac{d v_{cm}}{dx} = \theta_c = -\frac{12}{E t_m h_m^3} (- x - \frac{x^2}{2 L_a}) + C \tag{7-38}$$

$$v_{cm} = \frac{12}{E t_m h_m^3} (\frac{x^2}{2} - \frac{x^3}{6 L_a}) + Cx + D \tag{7-39}$$

at $x = 0$, $v_{cm} = 0$. Therefore $D = 0$ (7-40)

at $x = - L_a$, $v_{cm} = 0$. Hence (7-41)

$$C = -\frac{4 L_a}{E t_m h_m^3} \tag{7-42}$$

$$\Theta_c = \frac{12}{E t_m h_m^3}\left(-x - \frac{x^2}{2 L_a}\right) - \frac{4 L_a}{E t_m h_m^3} \tag{7-43}$$

at x = o

$$\Theta_c = -\frac{4 L_a}{E t_m h_m^3} \tag{7-44}$$

$$v^1_{cm} = \frac{12}{E t_m h_m}\left(-\frac{x^2}{2} - \frac{x^3}{6 L_a}\right) - \frac{4 L_a x}{E t_m h_m^3}$$

$$v^1_{cm} = -\frac{12}{E t_{cm} h_{cm}^3}\left(\frac{x^3}{6 L_a} + \frac{x^2}{2} + \frac{L_a x}{3}\right) \tag{7-45}$$

CHAPTER VIII

PLATE m OF SPAN ADC IS MADE CONTINUOUS

Equation (3-1) states

$$M_{om} = \frac{\theta_{am} - \theta_{bm}}{\theta_{cm} - \theta_{dm}} \qquad (3-1)$$

Now

$$\theta_c - \theta_d = \frac{-4 L_a}{E\, t_{am} h_{am}^3} - \frac{4 L_b}{E\, t_{bm} h_{bm}^3} \qquad (8-1)$$

$$\theta_a - \theta_b = \frac{1}{2 E\, t_{am} h_{am}^3} \left[S_{am} L_a^3 + b_{am} (T^1_{a(m-1)} + T^1_{am}) L_a^3 \right] \qquad (8-2)$$

$$+ \frac{1}{2 E\, t_{bm} h_{bm}^3} \left[S_{bm} L_b^3 - b_{bm} (T^1_{b(m-1)} + T^1_{bm}) L_b^3 \right]$$

Equations (8-1) and (8-2) are substituted in equation (3-1) and the redundant moment M_{om} is found.

$$M_{om} = - \frac{\theta_{am} - \theta_{bm}}{\theta_{cm} - \theta_{dm}} = \frac{\dfrac{1}{2 E\, t_{am} h_{am}^3} \left[S_{am} L_a^3 + b_{am} (T^1_{a(m-1)} + T^1_{am}) L_a^3 \right]}{\dfrac{4 L_a}{E\, t_{am} h_{am}^3} + \dfrac{4 L_b}{E\, t_{bm} h_{bm}^3}}$$

$$+ \frac{\dfrac{1}{2 E\, t_{bm} h_{bm}^3} \left[S_{bm} L_b^3 - b_{bm} (T^1_{b(m-1)} + T^1_{bm}) L_b^3 \right]}{\dfrac{4 L_a}{E\, t_{am} h_{am}^3} + \dfrac{4 L_b}{E\, t_{bm} h_{bm}^3}} \qquad (8-3)$$

Since v^1_{dm} is the deflection due to one unit moment applied at B, then

$$v_{dm} = v^1_{dm} M_o \qquad (8\text{-}4)$$

$$v_{dm} = M_{om} \frac{12}{E\, t_{bm} h^3_{bm}} \left(\frac{x^3}{6 L_b} - \frac{x^2}{2} + \frac{L_b x}{3} \right) \qquad (8\text{-}5)$$

Similarly

$$v_{cm} = M_{om} \frac{12}{E\, t_{am} h^3_{am}} \left(\frac{x^3}{6 L_a} - \frac{x^2}{2} + \frac{L_a x}{3} \right) \qquad (8\text{-}6)$$

The deflection of the continuous plate m for x larger than zero would consist of the sum of v_{bm} and v_{dm}. While for x smaller than zero the deflection would consist of the sum of v_{am} and v_{cm}. Hence

for $x > o$ $\qquad v_m = v_{bm} + v_{dm}$

$$v_m = - \frac{1}{2\, E\, t_{bm} h^3_{bm}} \left[\left(x^4 - 2 L_b x^3 + L_b^3 x \right) \left(S_m - h_m \left\langle T^1_{b(m-1)} + T^1_{bm} \right\rangle \right) \right]$$

$$+ M_o \frac{12}{E\, t_{dm} h^3_{dm}} \left(\frac{x^3}{6 L_b} - \frac{x^2}{2} + \frac{L_b x}{3} \right) \qquad (8\text{-}7)$$

for $x < o$ $\qquad v_m = v_{am} + v_{cm}$

$$v_m = \frac{-1}{2\, E\, t_{am} h^3_{am}} \left[\left(x^4 + 2 L_a x^3 - L_a^3 x \right) \left(S_{am} + h_{am} \left\langle T^1_{a(m-1)} + T^1_{am} \right\rangle \right) \right]$$

$$+ M_{om} \frac{12}{E\, t_{am} h^3_{am}} \left(\frac{x^3}{6 L_a} + \frac{x^2}{2} + \frac{L_a x}{3} \right) \qquad (8\text{-}8)$$

Similarly stress equations could be written for the continuous plate m. These equations would be too complicated as it is seen in the deflection curve equations above. For computation it is easier to solve the equations for the simply supported plates and then regroup the numerical answers. Hence it is unnecessary to give the stress equations for the continuous plates in this chapter.

CHAPTER IX

JOINT DISPLACEMENTS

The membrane theory outlined in the previous chapters is rather imperfect. Hence, a bending theory is outlined in the following pages in order to get a more realistic stress analysis.

The procedure used is identical with the usual analysis of statically indeterminate structures. The rigid connections of the plate strips are replaced by piano hinges which can transmit the shear T_m, but which cannot transmit plate bending moments M_y from one strip to the next. These moments are later chosen as the redundant quantities. The resulting hinged system is statically undeterminate because of the edge shears. The superscript (0) is going to be used to indicate the load action in the piano hinged system.

Fig. 19 shows a cross section (x = constant) of the structure with plate strips m - 1, m, m + 1. These plates are considered to be connected to each other by hinges; a study of their deformation is made. The deflection curve equations for these plates have been developed in Chapters IV to VII for simply supported plates and in Chapter VIII for continuous plates.

The corner m in Fig. 19 is at the same time a point of two strips. According to plate m point m has to undergo the displacement v_m while the strip m + 1 requires that the same point have displacement v_{m+1}. To satisfy

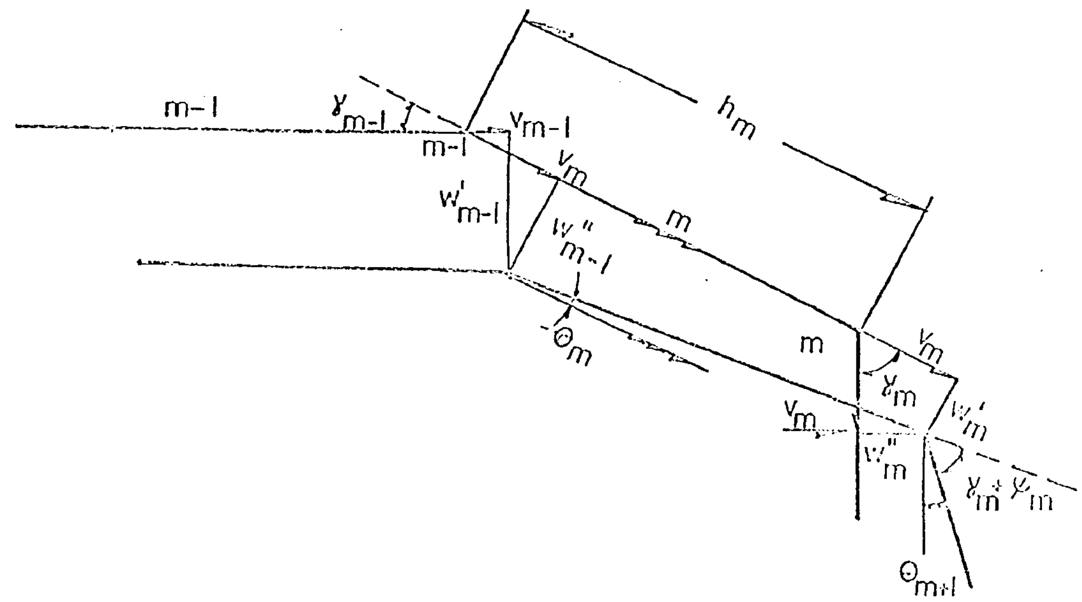


FIG. 19. NORMAL DISPLACEMENTS w'_m & w''_m

both requirements arbitrary displacements normal to each plate strip are added at point m. These normal displacements are justified only if the cross section remains straight and the plates are thin and long enough not to offer any substantial resistance to lengthwise bending and twisting. Hence we add to v_m the normal displacement w'_m at the corner m and the displacement w''_{m-1} at the corner m-1. Similarly to v_{m+1} we add the normal displacement w''_m at corner m and w'_{m+1} at point m+1.

By simple trigonometry we find the following relations:

$$\begin{aligned} w'_m \sin \gamma_m &= v_{m+1} - v_m \cos \gamma_m, \\ w''_m \sin \gamma_m &= v_{m+1} \cos \gamma_m - v_m. \end{aligned} \tag{9-1}$$

The bending moments M_x and M_{xy} are small enough to be neglected. On the other hand, the bending moment My is rather large and it must be solved.

Because of the displacements w''_{m-1} and w'_m of its ends, the straight line m-1, m rotates clockwise by the angle

$$\Theta_m = \frac{w'_m - w''_{m-1}}{h_m}, \tag{9-2}$$

and the line m, m+1 rotates by a similar angle Θ_{m+1}. The difference of the two angles Θ_{m+1} and Θ_m is the angle ψ_m. Hence

$$\psi_m = \Theta_{m+1} - \Theta_m = \frac{w'_{m+1} - w''_m}{h_{m+1}} - \frac{w'_m - w''_{m-1}}{h_m} \tag{9-3}$$

Equation (9-1) may be used to express w', w'' in terms of the deflection v of the continuous plate. Then equations (8-7) and (8-8) may be used to express

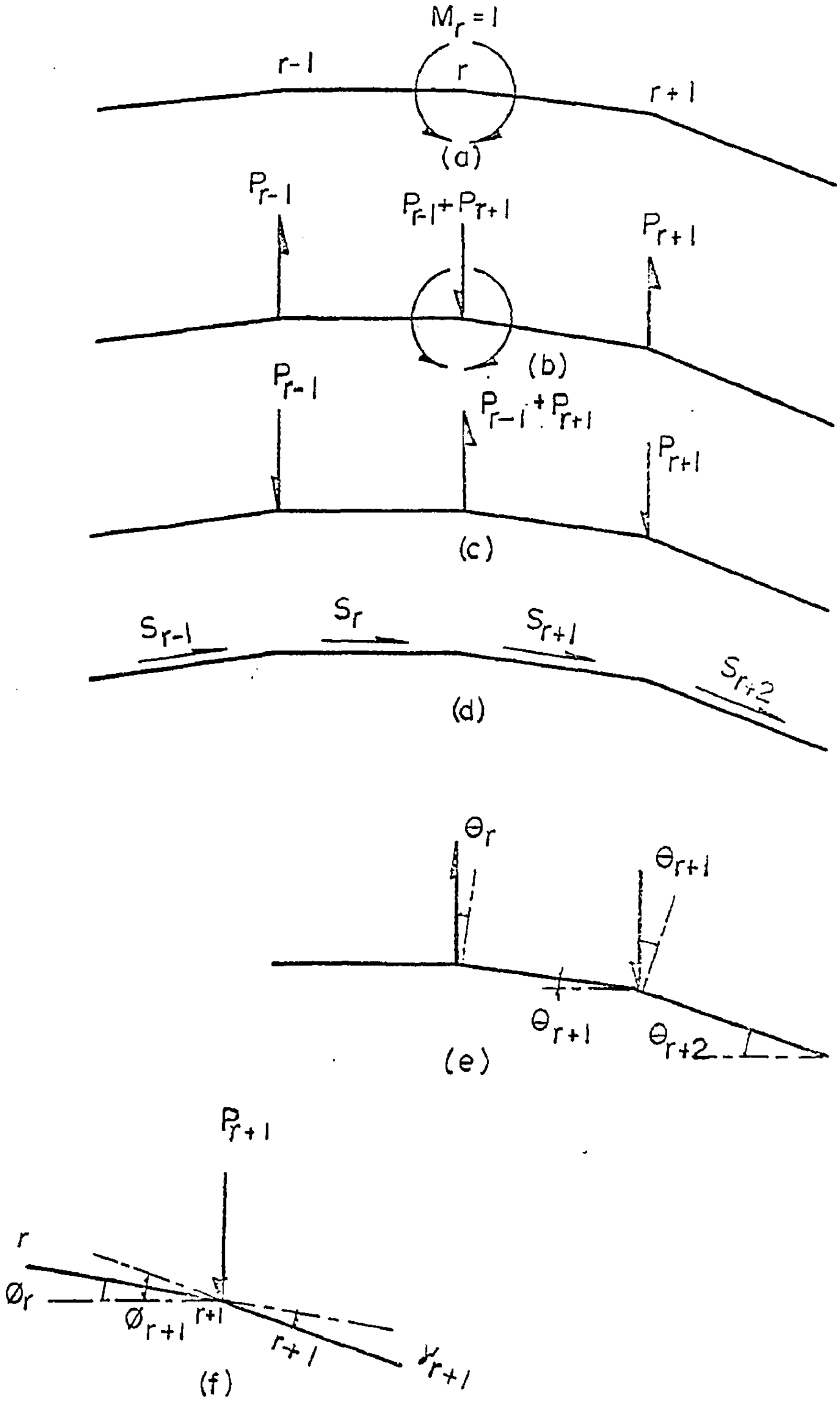


FIG. 20. REDUNDANT MOMENT M_r

v in terms of the loads S and the edge shears T that go with them in the hinged structure. We shall designate the ensuing deformation in eqs. (8-7), (8-8), (9-2), (9-3) by the superscript (0), i.e. $v_m^{(0)}$, $\Theta_m^{(0)}$, $\varphi_m^{(0)}$. They are the deformations of the principal system under the given load.

In the actual structure the strips are not connected by piano hinges but are so fixed that a relative rotation φ_m cannot take place. It is prevented by bending moments, which deform the straight cross sections shown in Fig. 19 into gentle curves whose tangents meet at the same angles γ_m as do the strips in the unstressed structure.

The moment M_y transmitted across the edge m = r from the strip r to the strip r + 1 is denoted by M_r and it depends on x.

We now have to study the internal force system set up by applying the moment $M_r = 1$ as an external unit load to the hinged system. The moment shown in Fig. 20(a) is replaced by the two loads in Figs. 20 b,c. The forces and moments shown in Fig. 20(b) are in local equilibrium.

The forces in Fig. 20(c) are applied as loads to the entire structure. Taking moments about r

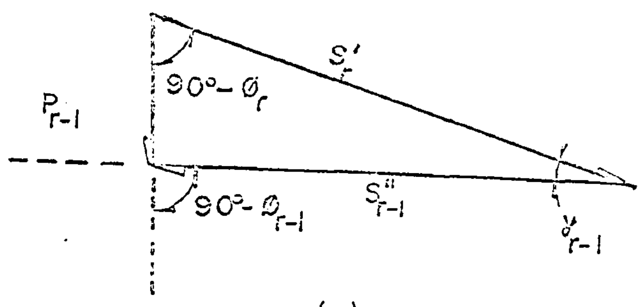
$$P_{r-1}\,(h_r \cos \Theta_r) = 1 \tag{9-4}$$

$$P_{r+1}\,(h_{r+1} \cos \Theta_{r+1}) = 1 \tag{9-5}$$

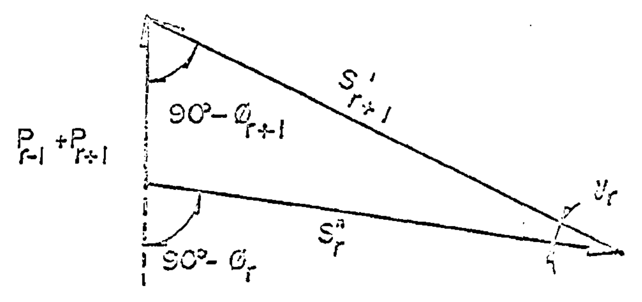
Hence

$$P_{r-1} = \frac{1}{h_r \cos \varphi_r} \tag{9-6}$$

$$P_{r+1} = \frac{1}{h_{r+1} \cos \Theta_{r+1}} \tag{9-7}$$



(a)



(b)

FIG. 21. REDUNDANT MOMENT M_r RESOLVED INTO FORCES ACTING IN PLANES OF PLATES

The vertical load P can be resolved into loads S acting in the planes of the corresponding plate strips. From Fig. 21(a)

$$S^{(r)}_{r-1} = -P_{r-1}\,\frac{\cos\varphi_r}{\sin\gamma_{r-1}}$$

$$S^{(r)}_{r-1} = -\,\frac{1}{h_r\,\sin\gamma_{r-1}} \qquad (9\text{-}8)$$

$$S^{(r)'}_{r} = -P_{r-1}\,\frac{\sin\theta_{r-1}}{\sin\gamma_{r-1}} \qquad (9\text{-}9)$$

The superscript (r) indicates the edge where the moment $M_r = 1$ is applied.

From Fig. 21(b) we obtain the following:

$$\frac{S^{(r)''}_{r}}{\sin(90^{\circ}-\theta_{r+1})} = \frac{P_{r-1}+P_{r+1}}{\sin\gamma_r}$$

$$S^{(r)''}_{r} = (P_{r-1}+P_{r+1})\,\frac{\cos\varphi_{r+1}}{\sin\gamma_r} \qquad (9\text{-}10)$$

The expressions for P_{r-1} and P_{r+1} are substituted into equation (9-10). After simplification

$$S^{(r)''}_{r} = -\,\frac{\cos\varphi_{r+1}}{h_r\cos\varphi_r\,\sin\gamma_r} - \frac{1}{h_{r+1}\,\sin\gamma_r} \qquad (9\text{-}11)$$

$$S^{(r)}_{r} = -S^{(r)'}_{r} - S^{(r)''}_{r} \qquad (9\text{-}12)$$

Equations (9-9) and (9-11) are substituted into (9-12).

$$S^{(r)}_{r} = \frac{1}{h_{r+1}\,\sin\gamma_r} + \frac{1}{h_r\cos\varphi_r}\left(\frac{\cos\varphi_{r-1}}{\sin\gamma_{r-1}} + \frac{\cos\varphi_{r+1}}{\sin\gamma_r}\right) \qquad (9\text{-}13)$$

Fig. 21(b) is used again to solve for $S^{(r)1}_{r+1}$

$$-\frac{S^{(r)1}_{r+1}}{\sin(90^{o}+\phi_{r})} = \frac{P_{r-1}+P_{r+1}}{\sin\gamma_{r}}$$

$$S^{(r)'}_{r+1} = -\left(\frac{1}{h_{r}\cos\phi_{r}}+\frac{1}{h_{r+1}\cos\phi_{r+1}}\right)\frac{\cos\phi_{r}}{\sin\gamma_{r}}$$

$$S^{(r)'}_{r+1} = \frac{-1}{h_{r}\sin\gamma_{r}} - \frac{\cos\phi_{r}}{h_{r+1}\cos\phi_{r+1}\sin\gamma_{r}} \qquad (9\text{-}14)$$

From Fig. 21(c)

$$\frac{S^{(r)''}_{r+1}}{\sin(90^{o}+\phi_{r+2})} = \frac{P_{r+1}}{\sin\gamma_{r+1}}$$

$$S^{(r)''}_{r+1} = \frac{-1}{h_{r+1}\cos\phi_{r+1}}\;\frac{\cos\phi_{r+2}}{\sin\gamma_{r+1}} \qquad (9\text{-}15)$$

Combining equations (9-14) and (9-15)

$$S^{(r)}_{r+1} = -\frac{1}{h_{r}\sin\gamma_{r}} - \frac{1}{h_{r+1}\cos\phi_{r+1}}\left(\frac{\cos\phi_{r}}{\sin\gamma_{r}}+\frac{\cos\phi_{r+2}}{\sin\gamma_{r+1}}\right) \qquad (9\text{-}16)$$

Again from Fig. 21(c)

$$\frac{S^{(r)}_{r+2}}{\sin(90^{o}-\phi_{r+1})} = \frac{P_{r+1}}{\sin\gamma_{r+1}}$$

$$S^{(r)}_{r+2} = \frac{1}{h_{r+1}\sin\gamma_{r+1}} \qquad (9\text{-}17)$$

The resolved loads $S^{(r)}_{r-1}$, $S^{(r)}_{r}$, $S^{(r)}_{r+1}$, $S^{(r)}_{r+2}$ acting on the plane of their respective plate strips are the result of the force system shown in Fig. 20(c). They may be summarized in the following way.

$$\left.\begin{aligned}
S^{(r)}_{r-1} &= -\frac{1}{h_r \sin\gamma_{r-1}} \\
S^{(r)}_{r} &= \frac{1}{h_{r+1}\sin\gamma_r} + \frac{1}{h_r\cos\psi_r}\left(\frac{\cos\psi_{r-1}}{\sin\gamma_{r-1}} + \frac{\cos\psi_{r+1}}{\sin\gamma_r}\right) \\
S^{(r)}_{r+1} &= -\frac{1}{h_r \sin\gamma_r} - \frac{1}{h_{r+1}\cos\psi_{r+1}}\left(\frac{\cos\psi_r}{\sin\gamma_r} + \frac{\cos\psi_{r+2}}{\sin\gamma_{r+1}}\right) \\
S^{(r)}_{r+2} &= \frac{1}{h_{r+1}\sin\gamma_{r+1}}
\end{aligned}\right\} \quad (9\text{-}18)$$

Equations (9-18) are substituted into equation (4-19), which will then yield the set of edge shears $T^{(r)}_m$ that goes with the unit load $M_r = 1$ when section BC is considered simply supported. Hence

$$\left.\begin{aligned}
&-\frac{4T^{(r)1}_{r-1} + 2T^{(r)1}_{r-2}}{t_{r-1}h_{r-1}} - \frac{3T^{(r)1}_{r} + 4T^{(r)1}_{r-1}}{t_r h_r} = -\frac{3S^{(r)}_{r-1}}{t_{r-1}h^2_{r-1}} - \frac{3S^{(r)}_{r}}{t_r h^2_r} \\
&\frac{4T^{(r)1}_{r} + 2T^{(r)1}_{r-1}}{t_r h_r} - \frac{2T^{(r)1}_{r+1} + T^{(r)1}_{r}}{t_{r+1}h_{r+1}} = -\frac{3S^{(r)}_{r}}{t_r h^2_r} - \frac{3S^{(r)}_{r+1}}{t_{r+1}h^2_{r+1}} \\
&-\frac{4T^{(r)1}_{r+1} + 2T^{(r)1}_{r}}{t_{r+1}h_{r+1}} - \frac{2T^{(r)1}_{r+2} + 4T^{(r)1}_{r+1}}{t_{r+2}h_{r+2}} = -\frac{3S^{(r)}_{r+1}}{t_{r+1}h^2_{r+1}} - \frac{3S^{(r)}_{r+2}}{t_{r+2}h^2_{r+2}} \\
&-\frac{4T^{(r)1}_{r+2} + 2T^{(r)1}_{r+1}}{t_{r+2}h_{r+2}} - \frac{2T^{(r)1}_{r+3} + 4T^{(r)1}_{r+2}}{t_{r+3}h_{r+3}} = -\frac{3S^{(r)}_{r+2}}{t_{r+2}h^2_{r+2}} - \frac{3S^{(r)}_{r+3}}{t_{r+3}h^2_{r+3}}
\end{aligned}\right\} \quad (9\text{-}19)$$

$T^{(r)}$ is the shear caused along different edges due to the unit moment M_y applied at edge r.

The loads $S^{(r)}$ and the shears $T^{(r)}$ must now be introduced into equations (4-51) and (4-52) in order to find the set of angles $\Theta^{(r)}_{b(x=0)m}$ and the set of deflections $v^{(r)}_{b(x=\frac{L}{2})m}$.

At this point adjustments for continuity must be made. Use equations (6-47) and (6-48) to find the rotations $\Theta_{d(x=o)m}$ and the deflections $v_{d(x=\frac{L}{2})m}$ due to a unit moment $M^{(r)}_{o(x=o)m}$. To solve $M^{(r)}_{o(x=o)m}$ equation (8-1) is used as follows

$$M^{(r)}_{o(x=o)m} = \frac{\Theta^{(r)}_{a(x=o)m} - \Theta^{(r)}_{b(x=o)m}}{\Theta_{c(x=o)m} \quad \Theta_{d(x=o)m}} \qquad (8-20)$$

Now the deflections $v^{(r)}_m$ of the continuous plate m supported at A, B, C can be calculated by using equations (8-7) and (8-8). With these values of $v^{(r)}_m$ and equations (9-1), (9-2), (9-3) we find the angles $\psi^{(r)}_m$ by which the strips rotate with respect to each other in the hinges due to a unit moment $M_r = 1$ applied at edge r. All these computations must be made separately for every r from r = 1 to r = k-1.

The angle $\psi^{(r)}_m$ represents only the deformation pertaining to the forces shown in Fig. 20(c). The forces and moments of Fig. 20(b) cause additional movements in the hinges. Since this load is in local equilibrium it only causes bending of the strips r and r+1, leading to the rotations of the end tangents shown in Fig. 22. Instead of first writing the rotations caused by M_r at different hinges and then adding the effects of different such moments on the rotation at the hinge m, we may at once write the angle $w_m = w^1_m + w''_m$ caused by the action of moments M_{m-1}, M_{m1}, M_{m+1}. Hence, from a well known beam formula we can write

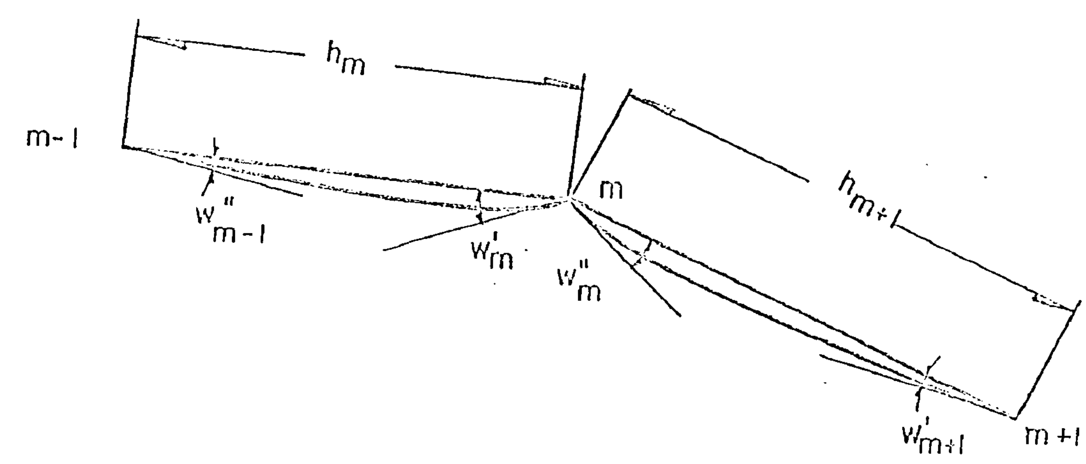


FIG. 22. DEFINITION OF THE ANGULAR DISPLACEMENTS w' & w''

$$w_m = \frac{h_m}{6\,K_m}\,(M_{m-1} + 2\,M_m) + \frac{h_{m+1}}{6\,K_{m+1}}\,(2M_m + M_{m+1}) \qquad (9\text{-}21)$$

Where K_m is the bending stiffness of the m th strip and is calculated from the equation

$$K_m = \frac{E\,t_m^3}{12(1-\mu^2)}. \qquad (9\text{-}22)$$

Now we may collect all the contributions to the relative rotation taking place at the hinge m. The given loads yield the value χ_m^0 obtained from equation (9-3) in the way already described. The moment M_r, at any arbitrary hinge r makes of contribution $\chi_m^r\,M_r$, and we have to write the sum of all these contributions from r = 1 to r = K - 1. Lastly, there is the contribution w_m of equation (9-21). The sum of all these is the relative rotation of the strips m and (m+1) in the hinge m, and since there is no hinge in the actual structure, the rotation must equal zero.

$$\sum_{r=1}^{K-1} \chi_m^{(r)} M_r + \frac{h_m}{6\,K_m}\,M_{m-1} + \frac{1}{3}\left(\frac{h_m}{K_m} + \frac{h_{m+1}}{K_{m+1}}\right) M_m$$

$$+ \frac{h_{m+1}}{6\,K_{m+1}}\,M_{m+1} + \chi_m^0 = 0 \qquad (9\text{-}23)$$

There are (K-1) such equations for the edges m = 1, 2 ------ (K-1), and these are just enough linear equations for the (K-1) unknown moments M_r. When these equations have been solved, we may easily retrace out steps and calculate all the stress resultants and displacements we desire.

CHAPTER X

ANALYTICAL SOLUTION OF EXPERIMENTAL PRISM

In this chapter a plate structure (Fig. 23) 24 foot long and supported at diagphrams A, B, C (Fig. 24) is analyzed for stresses and maximum deflections when the structure is subjected to a uniform load of 100 lb/ft applied at the upper edges b and c.

We consider the structure to be cut at the middle support thus consisting of two simply supported structures symmetrical about point B. Section BC is first analyzed. The vertical loads are resolved into components acting on the planes of the plate strips (Fig. 25). The longitudinal shears are calculated by substituting numerical values into equation (4-19). After simplification the

m	P_m lb/ft	ϕ_m	$\gamma_m = \phi_{m+1} - \phi_m$	$\cos \phi_m$	$\sin \gamma_m$	S'_m	S''_m	$S_m = S'_m + S''_{m-1}$
a	0	270	60	0	.866	0	0	0
b	100	330	30	.866	.500	-200	+173.2	-200
c	100	360	30	1.00	.500	-173.2	+200	0
d	0	30	60	.866	.866	0	0	+200
e	0	90	--	0	--	0	0	0

TABLE 10-1

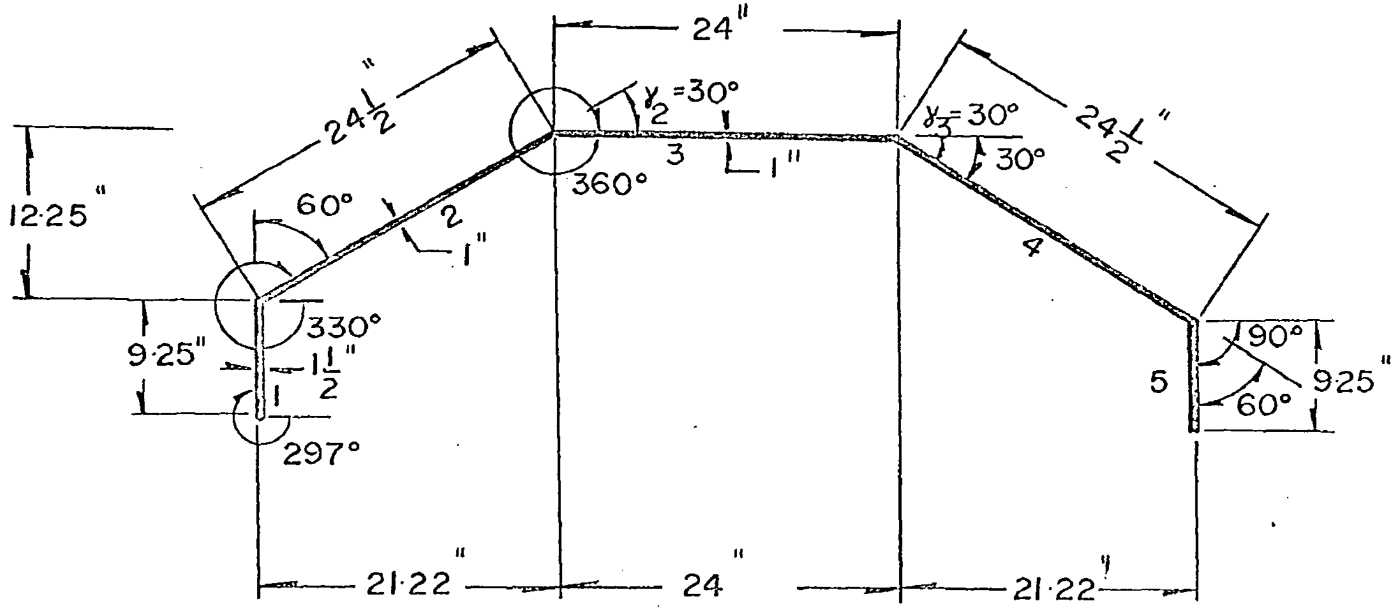


FIG. 23. DATA FOR ANALYTICAL PROBLEM

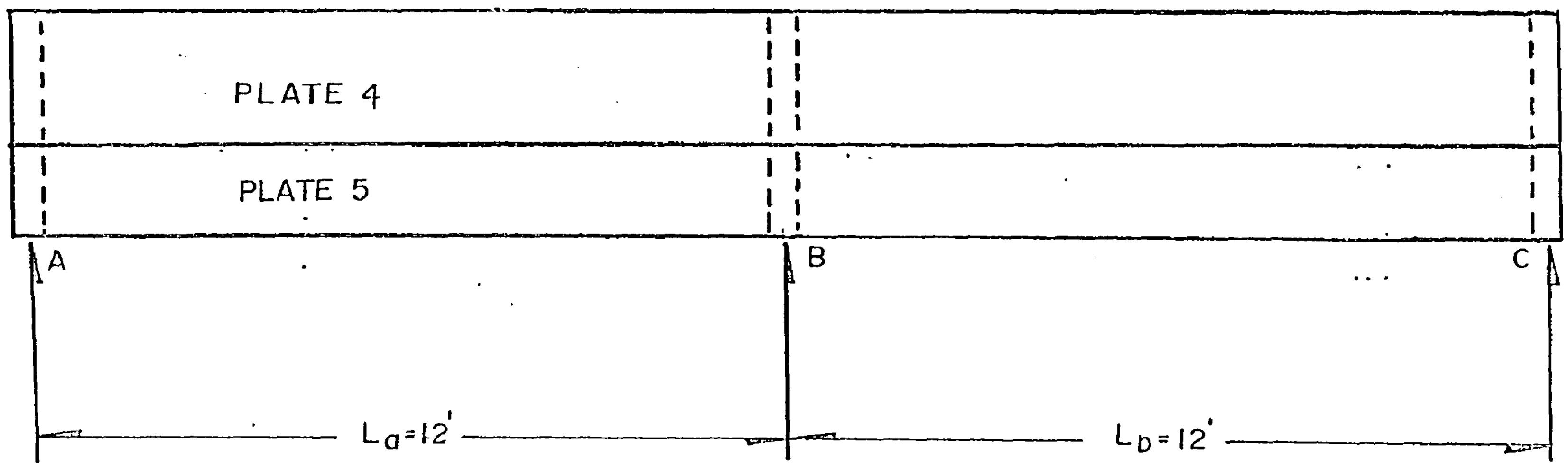


FIG. 24. DATA FOR ANALYTICAL PROBLEM

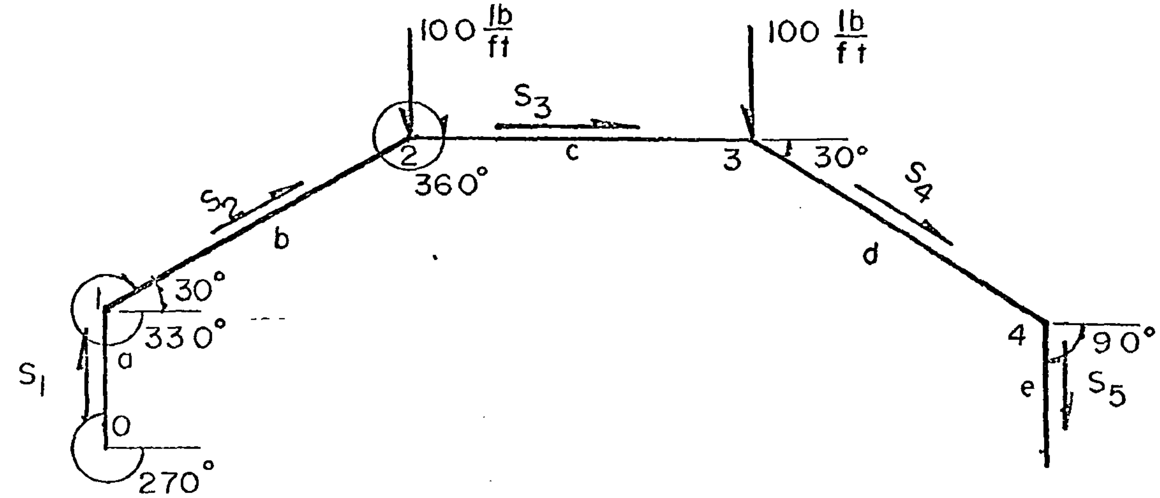


FIG. 25. LOADS RESOLVED INTO FORCES ACTING IN THE PLANES OF THE PLATES

four simultaneous shear equations are

$$32.52\ T^1_{b1} + 5.88\ T^1_{b2} = -1727.4$$
$$5.88\ T^1_{b1} + 23.76q\ T^1_{b2} + 6\ T^1_{b3} = -1727.4$$
$$6\ T^1_{b2} + 23.76\ T^1_{b3} + 5.88\ T^1_{b4} = 1727.4$$
$$5.88\ T^1_{b3} + 32.52\ T^1_{b4} = 1727.4$$

The shear constants T^1 produced by the vertical uniform load of 100 lb/ft applied at edges b and c are given in Table 10-2.

Long shear Constants	lb/ft^2
T^1_{b1}	-37.79
T^1_{b2}	-84.75
T^1_{b3}	+84.75
T^1_{b4}	+37.79

TABLE 10-2

Longitudinal Shear Constants for
Section BC

Because of symmetry the longitudinal shear constants T^1_a for section AB are equal and opposite to the shear constants T^1_b (Fig. 26a).

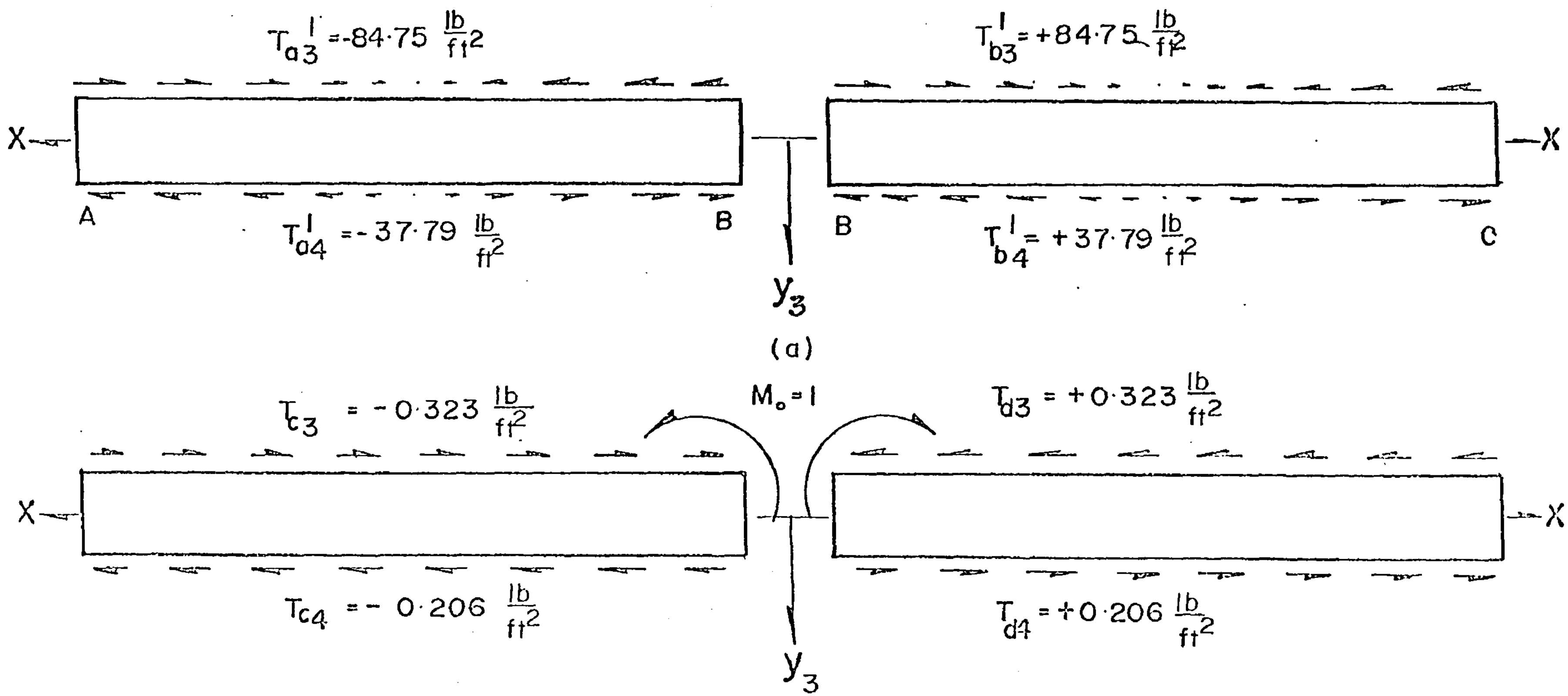


FIG. 26. LONGITUDINAL SHEARS

Long. Shear Constants	lb/ft^2
T^1_{a1}	+37.79
T^1_{a2}	+84.75
T^1_{a3}	-84.75
T^1_{a4}	-37.79

TABLE 10-3

Longitudinal Shear Constants for Section AB

Next, equation (4-51) is used to calculate the rotations of simply supported plates due to the uniform load of 100 lb/ft acting at edges b and c. Equation (4-51) yields the results shown in Table 10-4.

Rotation of Plates	Radians
θ_{b1}	-.001403
θ_{b2}	-.000194
θ_{b3}	0
θ_{b4}	+.000194
θ_{b5}	+.001403

TABLE 10-4

Rotation of Plates in Span BC Due to Uniform Load

Because of symmetry the rotations at $x = c$ for span AB are equal and opposite to those shown in Table 10-4.

Rotation of Plates	Radians
θ_{a1}	+.001403
θ_{a2}	+.000194
θ_{a3}	0
θ_{a4}	-.000194
θ_{a5}	-.001403

TABLE 10-5

Rotation of Plates in Span AB Due to Uniform Load

Equation (4-53) is used to find the deflections of plates between B and C. The results are shown in Table 10-6.

Deflections of Plates at $x=\frac{L_b}{2}$	Inches
v_{b1}	-.063075
v_{b2}	-.008736
v_{b3}	0
v_{b4}	+.008736
v_{b5}	+.063075

TABLE 10-6

Deflections of Plates at $x=\frac{L_b}{2}$ Due to Uniform Load

Because of symmetry about the y_m axis the deflections of plates at $x = \frac{L_n}{2}$ are equal to those shown in Table 10-6.

Deflections of Plates at $x=\frac{-L_a}{2}$	Inches
v_{a1}	-.063075
v_{a2}	-.008736
v_{a3}	0
v_{a4}	+.008736
v_{a5}	+.063075

TABLE 10-7

Deflections of Plates at $x=\frac{-L_a}{2}$ Due to Uniform Load

The two 12-foot sections (i.e. AB and BC) are now analyzed for shears, rotations, and deflections due to unit moments applied at end B of each different plate. When the unit moment is applied at end B of plate a, equations (6-15) will yield the following four simultaneous equations.

$$
\begin{aligned}
16.26\,T^a_{b1} - 5.88\,T^a_{b2} &= 6.73\\
5.88\,T^a_{b1} - 11.88\,T^a_{b2} + 6\,T^a_{b3} &= 0\\
6\,T^a_{b2} - 11.88\,T^a_{b3} + 5.88\,T^a_{b4} &= 0\\
5.88\,T^a_{b3} - 16.26\,T^a_{b4} &= 0
\end{aligned}
$$

Similarly, for a unit moment applied at end B of plate b.

$$
\begin{aligned}
16.26\,T^b_{b1} - 5.88\,T^b_{b2} &= 1.44\\
5.88\,T^b_{b1} - 11.88\,T^b_{b2} + 6\,T^b_{b3} &= -1.44\\
6\,T^b_{b2} - 11.88\,T^b_{b3} + 5.88\,T^b_{b4} &= 0\\
5.88\,T^b_{b3} - 16.26\,T^b_{b4} &= 0.
\end{aligned}
$$

For a unit moment applied at end B of plate c

$$16.26\ T^c_{b1} - 5.88\ T^c_{b2} = 0$$

$$5.88\ T^c_{b1} - 11.88\ T^c_{b2} + 6\ T^c_{b3} = -1.50$$

$$6\ T^c_{b2} - 11.88\ T^c_{b3} + 5.88\ T^c_{b4} = -1.50$$

$$5.88\ T^c_{b3} - 16.26\ T^c_{b4} = 0.$$

For a unit moment applied at end B of plate d

$$16.26\ T^d_{b1} - 5.88\ T^d_{b2} = 0$$

$$5.88\ T^d_{b1} - 11.88\ T^d_{b2} + 6\ T^d_{b3} = 0$$

$$6\ T^d_{b2} - 11.88\ T^d_{b3} + 5.88\ T^d_{b4} = -1.44$$

$$5.88\ T^d_{b3} - 16.26\ T^d_{b4} = -1.44.$$

For a unit moment applied at end B of plate e

$$16.26\ T^e_{b1} - 5.88\ T^e_{b2} = 0$$

$$5.88\ T^e_{b1} - 11.88\ T^e_{b2} + 6\ T^e_{b3} = 0$$

$$6\ T^e_{b2} - 11.88\ T^e_{b3} + 5.88 T^e_{b4} = 0$$

$$5.88\ T^e_{b3} - 16.26\ T^e_{b4} = -0.73.$$

The results of the above five sets of shear equations are tabulated in Table 10-6.

Unit M applied at B of plate	a	b	c	d	e
T_{b1}	.569	.206	.145	.072	.069
T_{b2}	.401	.323	.400	.199	.247
T_{b3}	.247	.199	.400	.323	.401
T_{b4}	.069	.072	.145	.206	.569

TABLE 10-6

Shear Forces in lb/ft for span BC Due to Unit Moments Applied At end B of Different Plates

Because of symmetry about the y_m axis the shears in the simply supported structure AB are equal and opposite [Fig. 26(a)] to those tabulated in Table 10-8.

Unit Moment Applied at End B of Plate	a	b	c	d	e
T_{a1}	-.559	-.206	-.145	-.072	-.089
T_{a2}	-.401	-.323	-.400	-.199	-.247
T_{a3}	-.247	-.199	-.400	-.323	-.401
T_{a4}	-.089	-.072	-.145	-.206	-.559

TABLE 10-9

Shear Forces in lb/ft for Span AB Due to Unit Moments Applied at End B of Different Plates

When a unit moment is applied at the end of a plate it causes that plate to rotate thus giving rise to deflections. The rotation of plates in span BC at B are found by substituting numerical values in Eequation (6-47) and are tabulated in Table (10-10).

Angles at x=o in Span BC	Radians
θ_{d1}	2.673×10^{-6}
θ_{d2}	2.158×10^{-7}
θ_{d3}	2.296×10^{-7}
θ_{d4}	2.158×10^{-7}
θ_{d5}	2.673×10^{-6}

TABLE 10-10

Rotations at B of Plates in Span BC Due to Unit Moments Applied at B

Because of symmetry the angles of corresponding plates in the simply supported structure in span AB are the same in magnitude but opposite in sign as those shown in Table 10-10. The set of angles $\Theta_{c(x=o)}$ is found by using equation (7-44).

Angles at x=o in Span AB	Radians
Θ_{c1}	-2.673×10^{-6}
Θ_{c2}	-2.158×10^{-7}
Θ_{c3}	-2.296×10^{-7}
Θ_{c4}	-2.158×10^{-7}
Θ_{c5}	-2.673×10^{-6}

Table 10-11
Rotations at B of Plates in Span AB Due to Unit Moments Applied at B.

The deflections at x = + ^{L}b/2 due to unit moment at x=0 are calculated by equation (6-48). The deflections at x = -^{L}a/2 can be calculated by equation (7-43). Because of symmetry about the y_{01} axis, however, the deflections are the same at both point x = ^{L}b/2 and point x = -^{L}a/2. Hence both results are tabulated in Table 10-12. At this point we must make adjustments for continuity. The redundant Moment M_c is found by substituting numerical values into equation (31). The results are shown in Table 10-13.

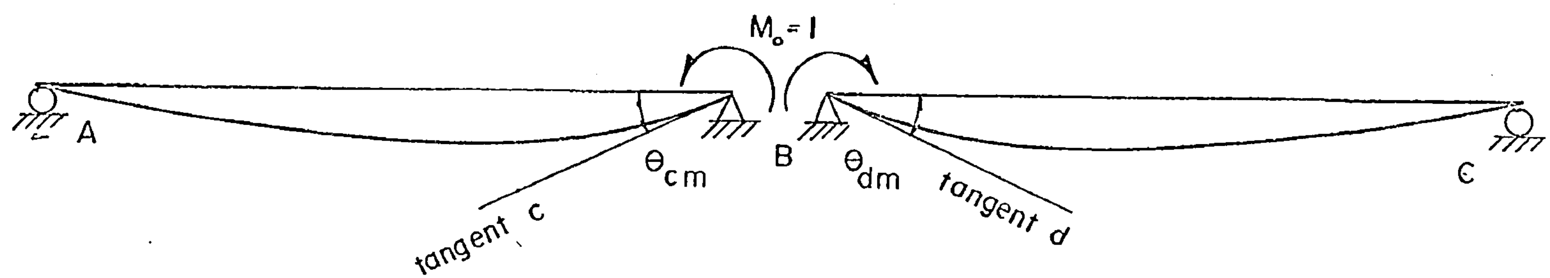


FIG. 27. ROTATIONS AT B DUE TO UNIT MOMENT AT B

Deflections at $x = {}^{-}L_a/2$		Deflections at $x = {}^{+}L_a/2$		Inches
v^1_{c1}	=	v^1_{d1}	=	7.215×10^{-5}
v^1_{c2}	=	v^1_{d2}	=	5.825×10^{-6}
v^1_{c3}	=	v^1_{d3}	=	6.1965×10^{-6}
v^1_{c4}	=	v^1_{d4}	=	5.825×10^{-6}
v^1_{c5}	=	v^1_{d5}	=	7.215×10^{-5}

TABLE 10-12

Deflections at Midspans Due to Unit Moments Applied at B

Redundant Moments		ft-lb
$M_{o,1}$	=	-524.21
$M_{o,2}$	=	-901.19
$M_{o,3}$	=	0
$M_{o,4}$	=	+901.19
$M_{o,5}$	=	+524.81

TABLE 10-13

Redundant Moments at Support B.

The deflections due to unit moments shown in Table 10-12 are now multiplied by the respective M_o (i.e. $v_{c1} = v^1_{c1} \times M_{o,1}$ etc.). The results are given in Table 10-14.

Deflections at $x={}^{-L}a/2$		Deflections at $x={}^{+L}b/2$		Inches
v_{c1}	=	v_{d1}	=	-.03787
v_{c2}	=	v_{d2}	=	-.005249
v_{c3}	=	v_{d3}	=	0
v_{c4}	=	v_{d4}	=	+.005249
v_{c5}	=	v_{d5}	=	+.03787

TABLE 10-14

Deflections at Midspans Due to the Redundant Moments Applied at B.

To find the deflections at midspans for the continuous system due to the uniform load of 100 lb/ft the deflections of Table 10-14 are subtracted from those of Table 10-6. The results are shown in Table 10-15.

Deflections at $x={}^{+L}b/2$ and $x={}^{-L}a/2$		Inches
v_1	=	.02521
v_2	=	-.003487
v_3	=	0
v_4	=	+.003487
v_5	=	+.02521

TABLE 10-15

Deflections at Midspans of the Continuous Structure
Under the Uniform Load of 100 lb/ft.

It must be remembered at this point that the deflections of Table 10-15 have been found under the assumption that the plates where connected to one another by piano hinges. We know, however, that the edges are rigidly fixed. Hence the theory of Chapter IX must be applied here to find the redundant moments M_y and study their effect.

Let us examine the deformation of the hinged system at $x = \frac{L_h}{2}$. Equations (9-1) are used to find the quantities w'_m, w''_m. From Fig. 28 we find the value γ_m.

γ_1	=	60°
γ_2	=	30°
γ_3	=	30°
γ_4	=	60°

TABLE 10-16

The values of w'_m and w''_m are tabulated below.

Displacements	Inches
w'_1	+.01052
w''_1	+.02710
w'_2	+.00604
w''_2	+.06974
w'_3	+.06974
w''_3	+.00604
w'_4	+.02710
w''_4	+.01052

TABLE 10-17

Normal Displacements (see Fig. 29)

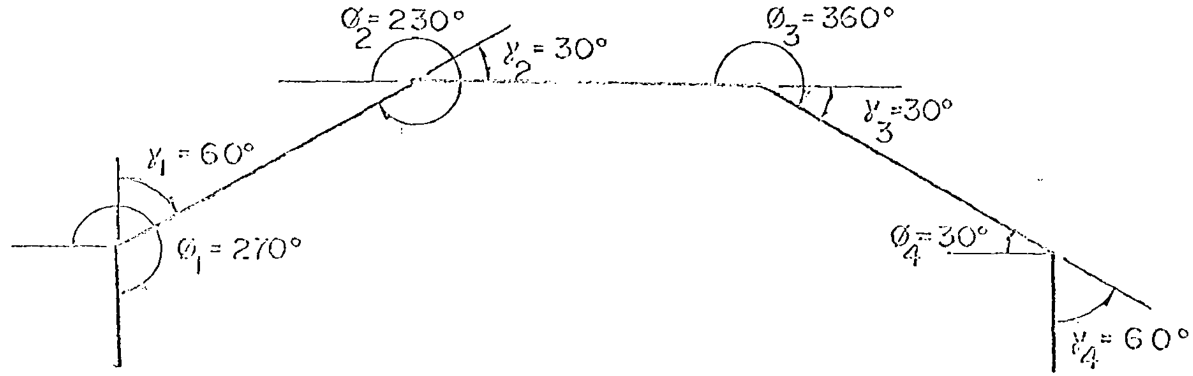


FIG. 28. ANGLES ϕ_m & γ_m

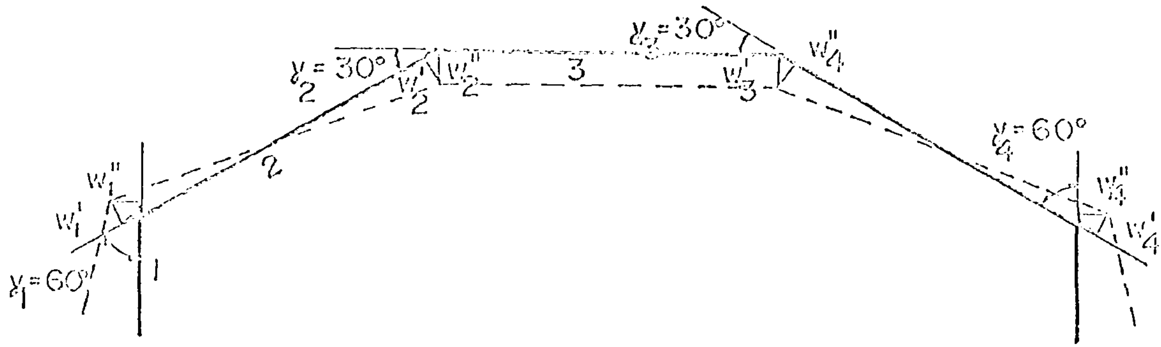


FIG. 29. NORMAL DISPLACEMENTS

Equation (9-2) is used to calculate the angles Θ_m. These values are tabulated in Table 10-18.

Angles	Radians
Θ_1	-8.6959×10^{-4}
Θ_2	-8.6959×10^{-4}
Θ_3	0
Θ_4	$+8.6959 \times 10^{-4}$
Θ_5	$+8.6959 \times 10^{-4}$

TABLE 10-18

Transversal Angles (see Fig. 19) at $x=+\frac{L_b}{2}$ and at $x=-\frac{L_a}{2}$

The angles ψ_m are calculated from equation (9-3) and are given in Table (10-19).

$\psi_1 =$	0
$\psi_2 =$	$+8.6959 \times 10^{-4}$ rad
$\psi_3 =$	$+8.6959 \times 10^{-4}$ rad
$\psi_4 =$	0

TABLE 10-19

Angles at $x=+\frac{L_b}{2}$ and at $x=-\frac{L_a}{2}$ (Fig. 29).

Since the strips are not connected by piano hinges in the actual structure, the relative rotation ψ_m cannot take place; it is prevented by the bending moments M_y. Let the moment M_y transmitted across the edge m = r from the strip r to the strip r + 1 be denoted by M_r. Now, we will study the internal force system set up by applying this moment with $M_r = 1$.

When moment $M_r = 1$ is applied at edge b of the structure (Fig. 30) equations (9-18) yield the results shown in Table 10-20. The superscript b above the S denotes the edge where the moments $M_r = 1$ is applied.

Tangential Loads	lb/ft
S_1^b	-.5656
S_2^b	+2.1312
S_3^b	-2.7117
S_4^b	+1
S_5^b	0

TABLE 10-20

Tangential Loads in the Planes of Four Strips (Fig. 30)
Due to Unit Moment $M_r = 1$ Applied at Edge b.

When moment $M_r = 1$ is applied at edge c of the structure (Fig. 31), equations (9-13) yield the results shown in Table 10-21.

Tangential Loads	lb/ft
S_1^c	0
S_2^c	-1.0000
S_3^c	+2.7117
S_4^c	-2.1312
S_5^c	+.5656

TABLE 10-21

Tangential Loads in the Planes of Four Strips (Fig. 31)
Due to Unit Moment $M_r = 1$ Applied at Edge c.

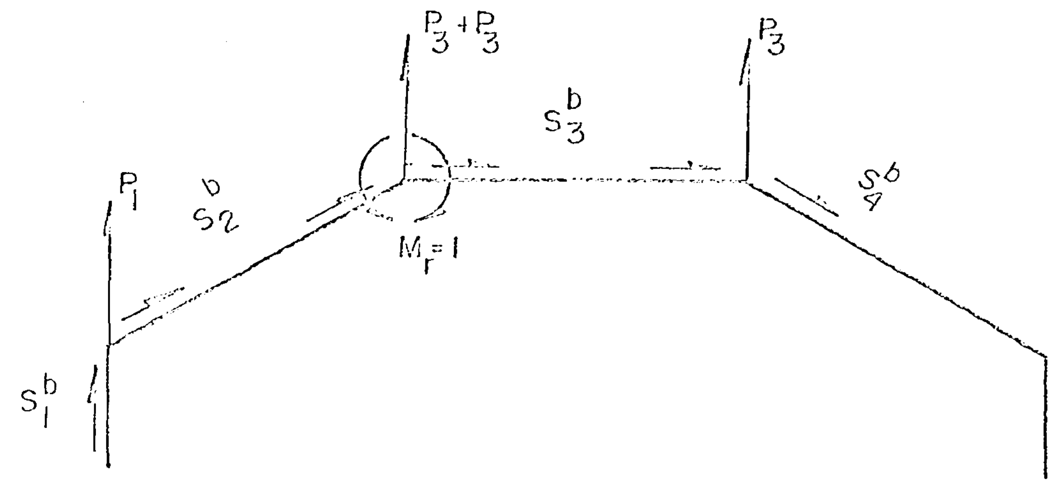


FIG. 30. REDUNDANT MOMENT $M_r = 1$
APPLIED AT EDGE b

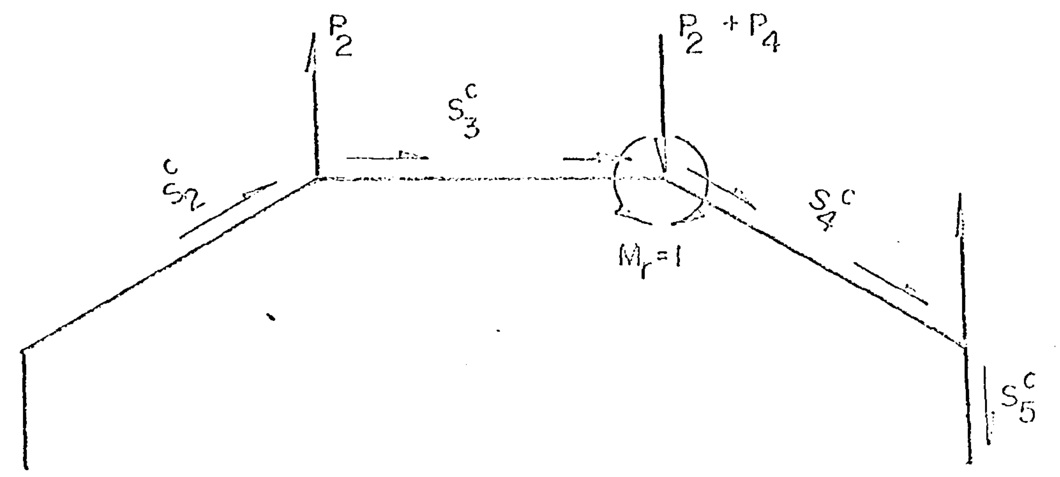


FIG. 31. REDUNDANT MOMENT M = I AT c RESOLVED INTO FORCES ACTING IN THE PLANES OF PLATES

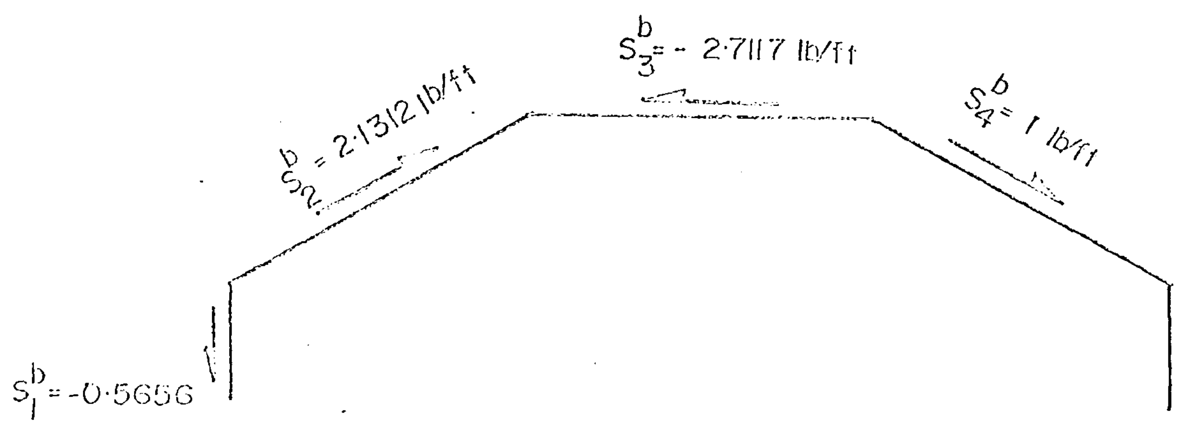


FIG. 32. NUMERICAL VALUES OF LOADS S^b

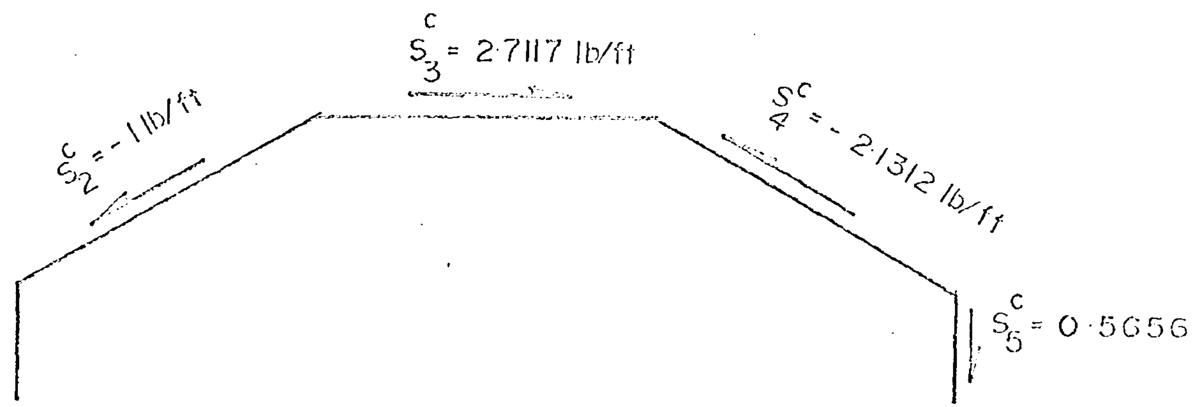


FIG. 33. NUMERICAL VALUES OF S^c

Next we may calculate shears T caused by unit moments $M_r = 1$ applied separately at edges b and c. For $M_r = 1$ (Fig. 32) applied at edge b, equations (4-19) yield the following simultaneous equations.

$$
\begin{aligned}
32.5116\, T_1^{(b)1} + 5.8775\, T_2^{(b)1} &= -4.440 \\
5.8775\, T_1^{(b)1} + 23.7550\, T_2^{(b)1} + 6\, T_3^{(b)1} &= -6.0 \\
6.0000\, T_2^{(b)1} + 23.755\, T_3^{(b)1} + 5.8775\, T_4^{(b)1} &= -15.769 \\
5.8775\, T_3^{(b)1} + 32.5116\, T_4^{(b)1} &= 8.6364
\end{aligned}
$$

For $M_r = 1$ (Fig. 33) applied at edge c, equations (4-19) yield the following simultaneous equations.

$$
\begin{aligned}
32.5116\, T_1^{(c)1} + 5.8775\, T_2^{(c)1} &= -8.6364 \\
5.8775\, T_1^{(c)1} + 23.7550\, T_2^{(c)1} + 6.000\, T_3^{(c)1} &= +15.7689 \\
6.0000\, T_2^{(c)1} + 23.7550\, T_3^{(c)1} + 5.8775\, T_4^{(c)1} &= +6.0 \\
5.8775\, T_3^{(c)1} + 32.5116\, T_4^{(c)1} &= 4.440
\end{aligned}
$$

The shear stresses due to unit moments applied transversely at edges b and c are tabulated in Table 10-22.

$T_1^{(b)1}$	-.1313
$T_2^{(b)1}$	-.0291
$T_3^{(b)1}$	-.7560
$T_4^{(b)1}$	+.4023
$T_1^{(c)1}$	-.4023
$T_2^{(c)1}$	+.7560
$T_3^{(c)1}$	+.0291
$T_4^{(c)1}$	+.1314

TABLE 10-22

Shear Stresses Due to Unit Moments Applied Transversely at Edges b and c

The loads $S^{(r)}$ and the edge shears $T^{(r)}$ must now be introduced into equations (4-51) and (4-53) to find rotations at x = o and deflections $x = \frac{L_b}{2}$. The results are shown in Tables (10-23 and (10-24) respectively. The structure at this point still consists of two separate simply supported sections.

$\theta_{b1}^{(b)}$	$+2.2345 \times 10^{-5}$ rad.
$\theta_{b2}^{(b)}$	$-.95510 \times 10^{-5}$
$\theta_{b3}^{(b)}$	$+.47169 \times 10^{-5}$
$\theta_{b4}^{(b)}$	$-.66893 \times 10^{-5}$
$\theta_{b5}^{(b)}$	$+1.49213 \times 10^{-5}$
$\theta_{b1}^{(c)}$	-1.49213×10^{-5}
$\theta_{b2}^{(c)}$	$+.66893 \times 10^{-5}$
$\theta_{b3}^{(c)}$	$-.47169 \times 10^{-5}$
$\theta_{b4}^{(c)}$	$+.95510 \times 10^{-5}$
$\theta_{b5}^{(c)}$	-2.2346×10^{-5}

TABLE 10-23

Rotations at x = o of plates in Span BC Due to $M_r = 1$ Applied at Edge b and $M_r = 1$ Applied at Edge c

Because of symmetry about the y_m axis the angles of rotation at x = o of plates in span AB due to the moment $M_r = 1$ separately applied at edges b and c are equal and opposite to the values appearing in Table 10-23.

Because of symmetry about the y_m axis the deflections at $x = \frac{L_b}{2}$ of plates in span AB due to the moment $M_r = 1$ separately applied at edges b and c are equal to the values of Table 10-24.

$v_{b1}^{(b)}$	+.001005 inches
$v_{b2}^{(b)}$	-.0004298
$v_{b3}^{(b)}$	+.0002123
$v_{b4}^{(b)}$	-.0003010
$v_{b5}^{(b)}$	+.0006714
$v_{b1}^{(c)}$	-.0006714 inches
$v_{b2}^{(c)}$	+.0003010
$v_{b3}^{(c)}$	-.0002123
$v_{b4}^{(c)}$	+.0004298
$v_{b5}^{(c)}$	-.001005

TABLE 10-24

Deflections at $x=\frac{L_b}{2}$ of Plates in Span BC Due to $M_x=1$ Applied at Edge b and $M_x=1$ Applied at Edge c

We can now make adjustments for continuity. Equation (3-1) becomes

$$M_{o(x=o)m}^{(r)} = \frac{\theta_{cm}^{(r)} - \theta_{dm}^{(r)}}{\theta_{cm} - \theta_{dm}} .$$

The angles $\theta_{cm}^{(r)}$ and $\theta_{dm}^{(r)}$ are given in Table 10-23, while the angles θ_{cm} and θ_{dm} are given in Tables 10-11 and 10-10 respectively. The resulting values of the redundant moments $M_{o(x=o)m}^{(r)}$ are tabulated in Table 10-25.

Moments at x=o	
$M^{b}_{o,1}$	+ 8.360 ft-lb
$M^{b}_{o,2}$	-44.259
$M^{b}_{o,3}$	+20.544
$M^{b}_{o,4}$	-30.998
$M^{b}_{o,5}$	+ 5.583
$M^{c}_{o,1}$	- 5.583
$M^{c}_{o,2}$	+30.998
$M^{c}_{o,3}$	-20.544
$M^{c}_{o,4}$	+44.259
$M^{c}_{o,5}$	- 8.360

TABLE 10-25

Redundant Moments $M^{(r)}_{o(x=o)m}$ Due to Moments $M_r=1$

The deflections in Table 10-12 are multiplied by the redundant moments in Table 10-25. The results are given in Table 10-26.

The deflections of the continuous beams ABC due to the moments M_r are now calculated. The deflections of Table 10-12 are subtracted from the deflections of Table 10-24. The results are the deflections of the plates of the continuous structure due to uniform moments $M_r = 1$ separately applied at edges b and c.

Deflections at $x = L_b/2$	Inches
$v^b_{d,1}$	$+6.0342 \times 10^{-4}$
$v^b_{d,2}$	-2.5790×10^{-4}
$v^b_{d,3}$	$+1.2734 \times 10^{-4}$
$v^b_{d,4}$	-1.8065×10^{-4}
$v^b_{d,5}$	$+4.0294 \times 10^{-4}$
$v^c_{d,1}$	-4.0294×10^{-4}
$v^c_{d,2}$	$+1.8065 \times 10^{-4}$
$v^c_{d,3}$	-1.2734×10^{-4}
$v^c_{d,4}$	$+2.5791 \times 10^{-4}$
$v^c_{d,5}$	-6.0342×10^{-4}

TABLE 10-26

Deflections at Midspan of BC Due to Redundant Moments $M_o^{(r)}$ Applied to Plates at B

The deflections at midspan of AB are the same as those shown in Table 10-26.

Deflections	Inches
$v_1^{(b)}$	-.0006029
$v_2^{(b)}$	-.0001719
$v_3^{(b)}$	+.0000850
$v_4^{(b)}$	-.0001204
$v_5^{(b)}$	+.0002685
v_1^c	-.0002685
v_2^c	+.0001204
v_3^c	-.0000850
v_4^c	+.0001719
v_5^c	+.0006029

TABLE 10-27

Deflections of the Plates of the Continuous Structure Due to

The deflections of Table 10-27 are substituted in equations (9-1) and the resulting w_s^i are shown in Table 10-28.

$w_1'^{(b)}$	.0009185 in.
$w_1''^{(b)}$	.0004814
$w_2'^{(b)}$	.0004677
$w_3''^{(b)}$	.0004910
$w_3'^{(b)}$	-.0003880
$w_3''^{(b)}$	-.0003785
$w_4'^{(b)}$	.0003795
$w_4''^{(b)}$	.0002940
$w_1'^{(c)}$	.0002940
$w_1''^{(c)}$	.0003795
$w_2'^{(c)}$	-.0003785
$w_2''^{(c)}$	-.0003880
$w_3'^{(c)}$	.0004910
$w_3''^{(c)}$	.0004677
$w_4'^{(c)}$	.0004814
$w_4''^{(c)}$	.0009185

TABLE 10-28

Normal Displacements to Each Plate Strip Due to Moments $M_r=1$
Applied at Edges b and c alternately

The values in Table 10-28 are substituted in equation (9-2) and the results are given in Table 10-29.

$\theta_1^{(b)}$	-5.592×10^{-7} rad.
$\theta_2^{(b)}$	-5.592×10^{-7}
$\theta_3^{(b)}$	-3.6625×10^{-5}
$\theta_4^{(b)}$	3.0939×10^{-5}
$\theta_5^{(b)}$	3.0939×10^{-5}
$\theta_1^{(c)}$	-3.0939×10^{-5}
$\theta_2^{(c)}$	-3.0939×10^{-5}
$\theta_3^{(c)}$	3.6625×10^{-5}
$\theta_4^{(c)}$	5.5918×10^{-7}
$\theta_5^{(c)}$	5.5918×10^{-7}

TABLE 10-29

Tranversal Rotation of Plate Strips at Midspans of Continuous Structure Due to M_r=1

The relative change of angle at the joints is given by equation (9-3). Values from Table 10-29 are substituted in equations 9-3 and the results are given in Table 10-30.

The values from Table 10-30 are substituted in equation (9-23) to find the redundant moments M_r. The results are given in Table 10-31.

$\psi_1^{(b)}$	=	0
$\psi_2^{(b)}$	=	-3.6066×10^{-5} rad.
$\psi_3^{(b)}$	=	$+6.7564 \times 10^{-5}$
$\psi_4^{(b)}$	=	0
$\psi_1^{(c)}$	=	0
$\psi_2^{(c)}$	=	6.7564×10^{-5}
$\psi_3^{(c)}$	=	-3.6066×10^{-5}
$\psi_4^{(c)}$	=	0

TABLE 10-30

Relative Angle Changes at Joints Due to $M_r=1$
Alternately Applied at Joints b and c

M_2	=	-11.0594 ft-lb
M_3	=	-11.0594 ft-lb

TABLE 10-31

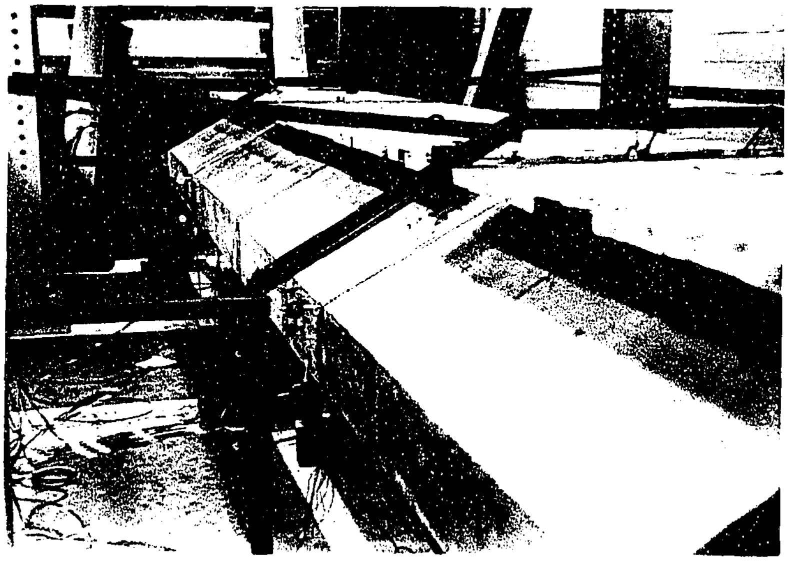
Redundant Moments Acting Uniformly along the
Edges b and c of the continuous structure

The deflections of Table 10-27 are multiplied by the redundant moments of Table 10-31 and the results are subtracted from the deflections given in Table 10-15. Hence the final deflections of the structure are given in Table 10-32.

v_1	=	-.010011 in.
v_2	=	-.002873 in.
v_3	=	0
v_4	=	+.002873 in.
v_5	=	+.0010011

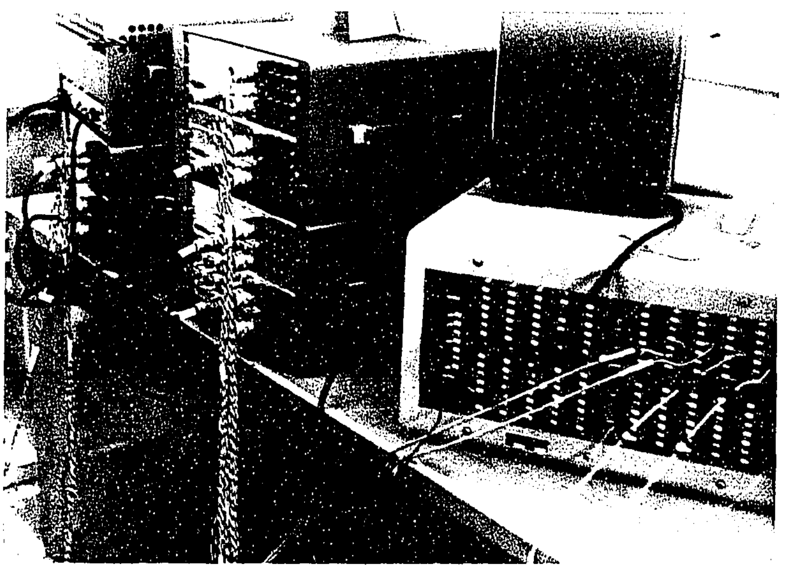
TABLE 10-32

Final Deflections of the Continuous Structure Due to a Uniform Load of 100 lb/ft Applied at Edge b and c.



(a)

Fig. 34 Three-lead wires connect the gages to the equipment. Electronic gages are connected to Budd instruments while the load cells are connected to Baldwin Lima instruments.

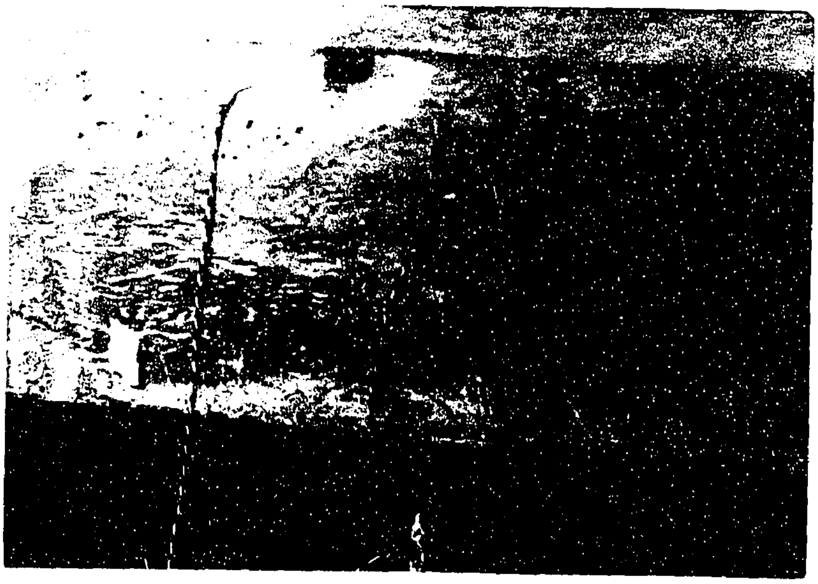


(b)

CHAPTER XI

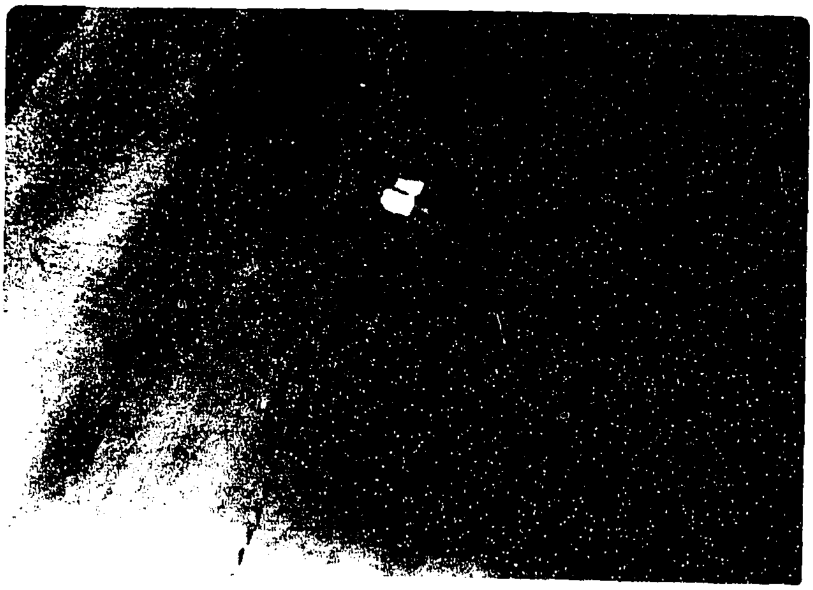
EXPERIMENTAL PROGRAM

In order to learn the actual pattern of deflections and of stress distribution of continuous folded plate structures a simple prismatic structure was made out of concrete ($f'_c = 6,000$ psi). The maximum size aggregate used in the concrete was 3/8". Reinforcing steel was used as shown in Fig. 47. The two spans were 12'-0" each from center of supports. The thickness of the three top plates was 1" while the two vertical plates containing 3/8" ⌀ bars were 1 1/2" thick (Fig. 47). To avoid concentration of stresses in lifting, the whole structure was constructed right on the load cells through the use of a platform which was removed after 28 days from the day of pouring. Concrete cylinders were tested after 28 days and the results are given in Fig. 51. Strain gages (SR4, type-C6-141-B with gage factor 2.08 $\pm$ 1/2%) were applied on the steel as well as on the concrete surface (Fig. 48). The results are given in Tables 11-1 and in Figs. 52-56. Gauge indicators were set up as shown in Figs. 49-50 and the results are given in Tables 11-2, 11-3 and in Fig. 57. Selected pictures are introduced in the following pages to avoid a long and tedious description of the experiment.

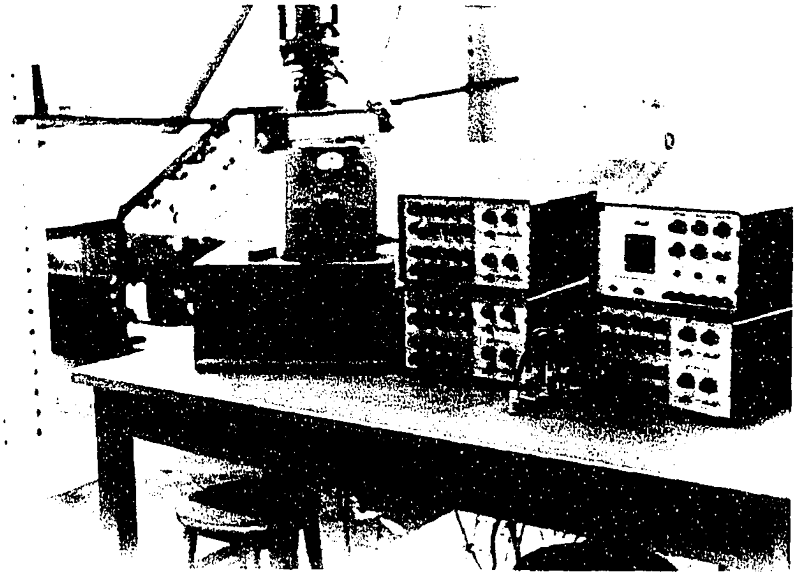


(a)

Fig. 35. Some of the strain gages are applied on the steel imbedded in the concrete, others are applied directly on the concrete surface. SR4 gages were used.

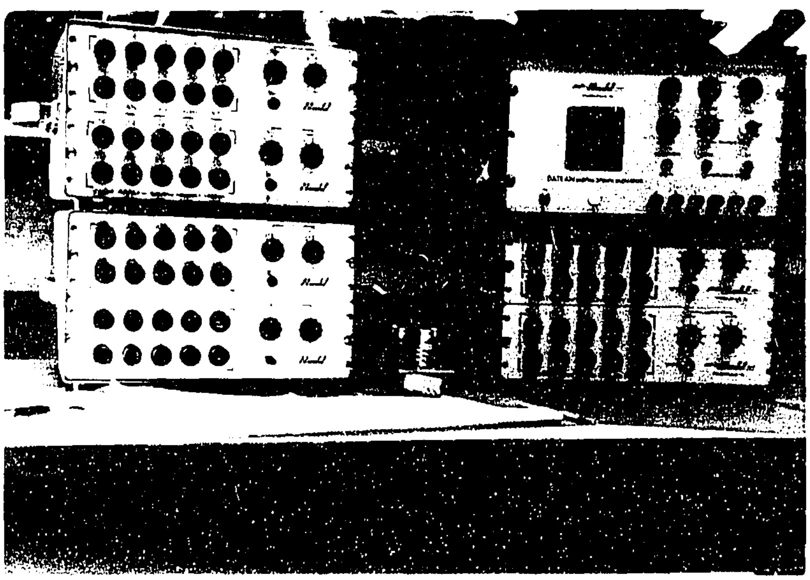


(b)

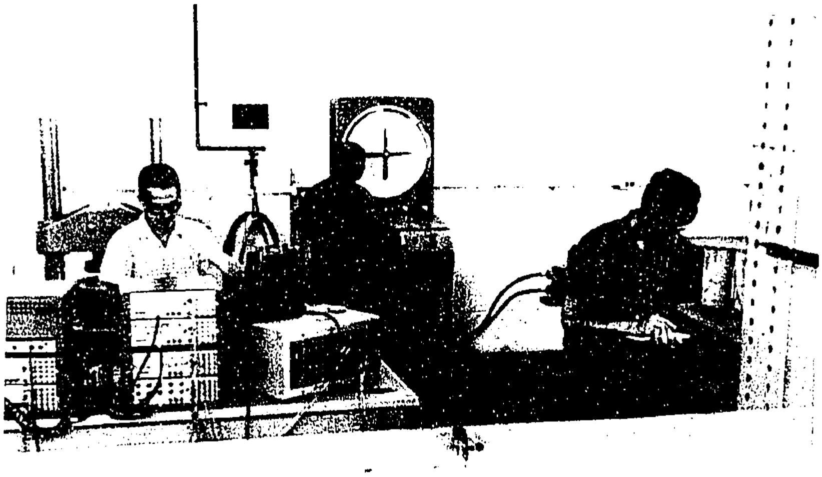


(a)

Fig. 36. Electronic equipment used to read strains of the strain gages.



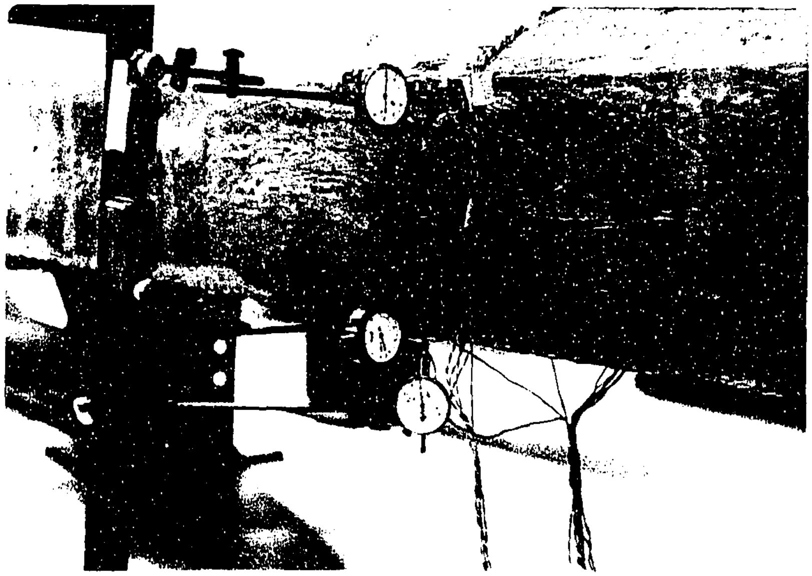
(b)



(a)

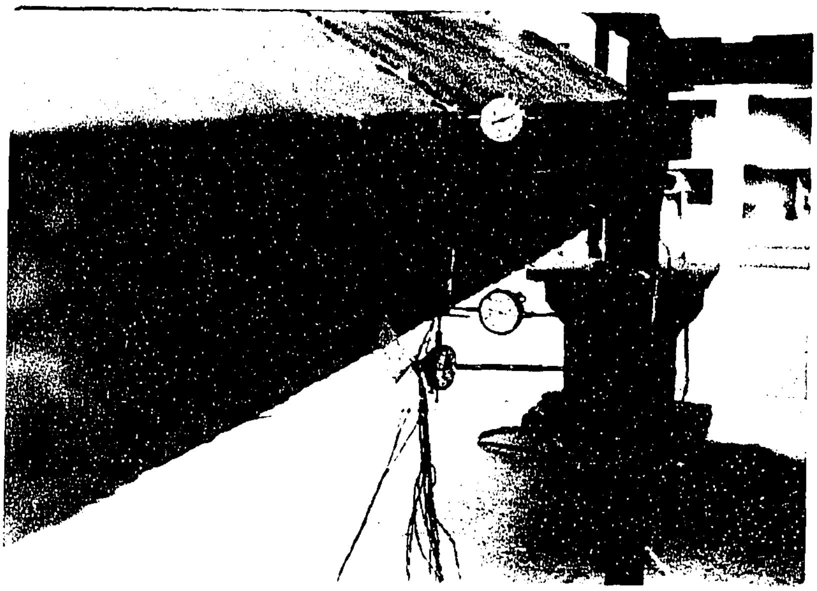
Fig. 37. Strain gage readings and dial indicator readings are taken simultaneously.

(b)



(a)

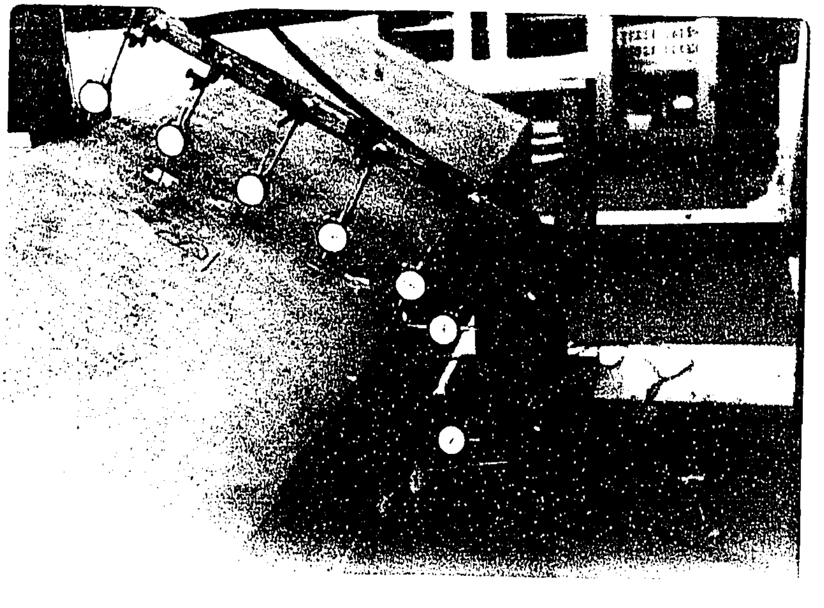
Fig. 38. Dial indicators are placed symmetrically around sections A and B to check whether the deflections of the symmetrical structure were the same on both sides. The results are given in Table 11-2.



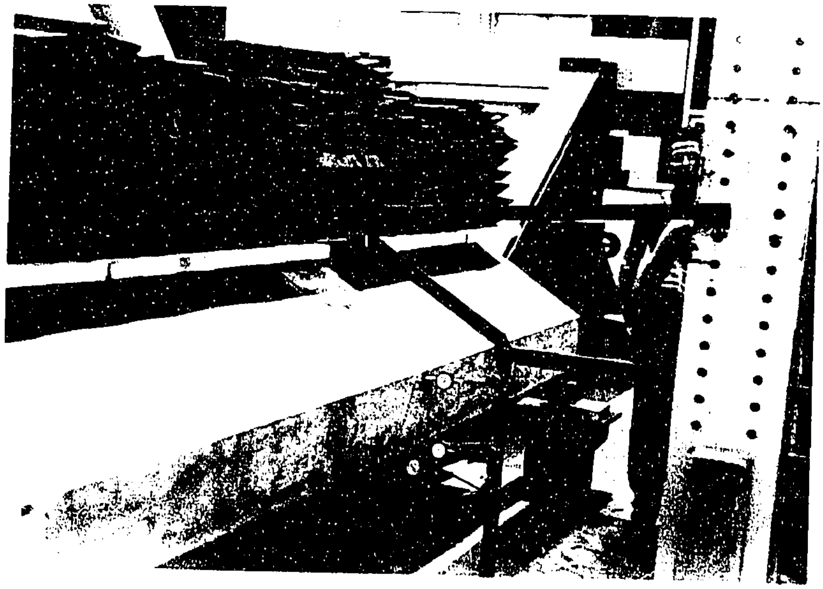
(b)

(a)

Fig. 39. Dial indicators are set up at midspans A and B to study the deflection pattern of the structure. Because of symmetry the dial indicators are placed only on one side of the structure. The results are given in Table 11-3.

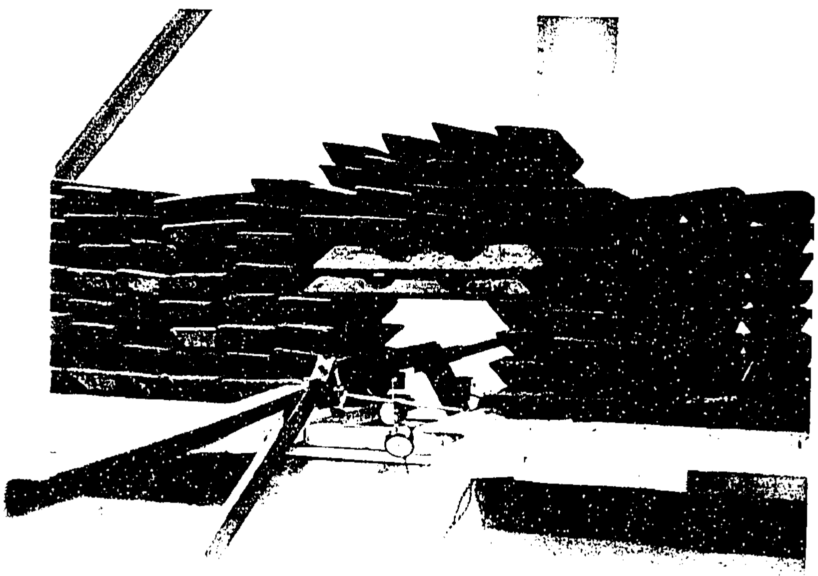


(b)

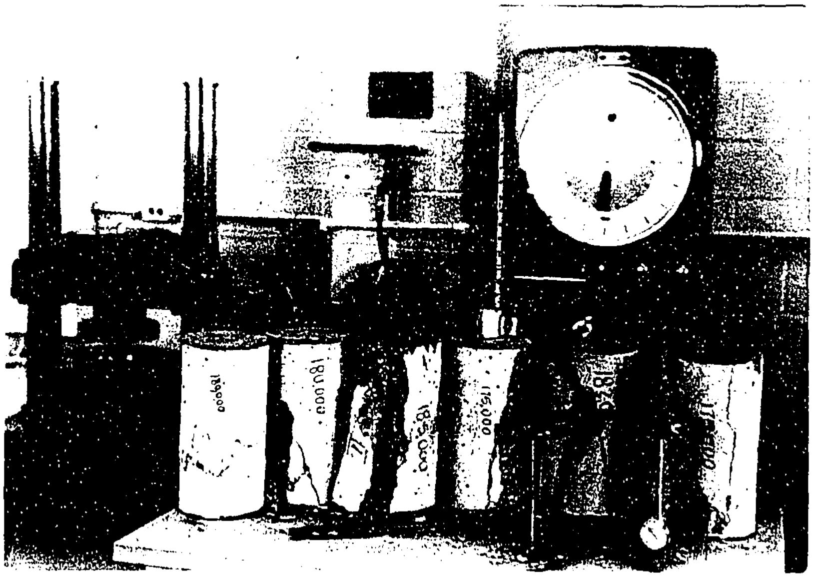


(a)

Fig. 40. The dial indicators are applied on stiff frames made out of 2" x 2" x 1/4" steel angles.

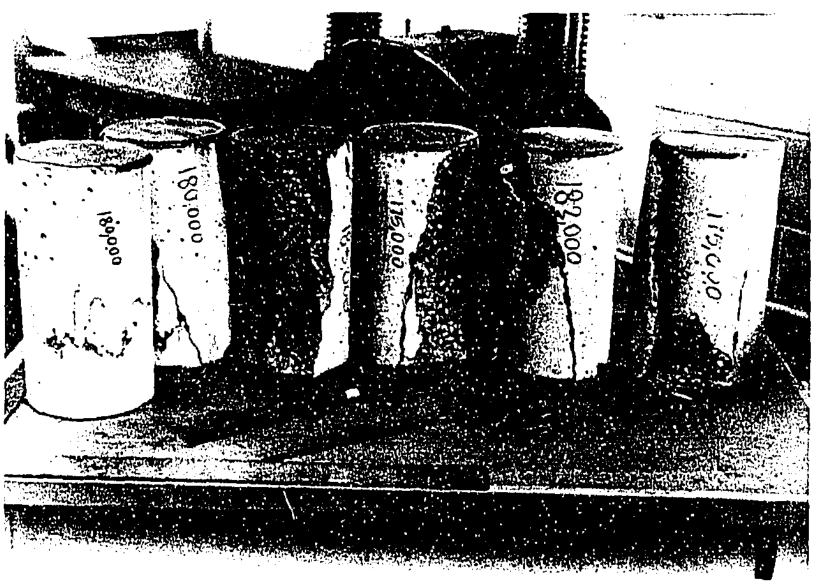


(b)

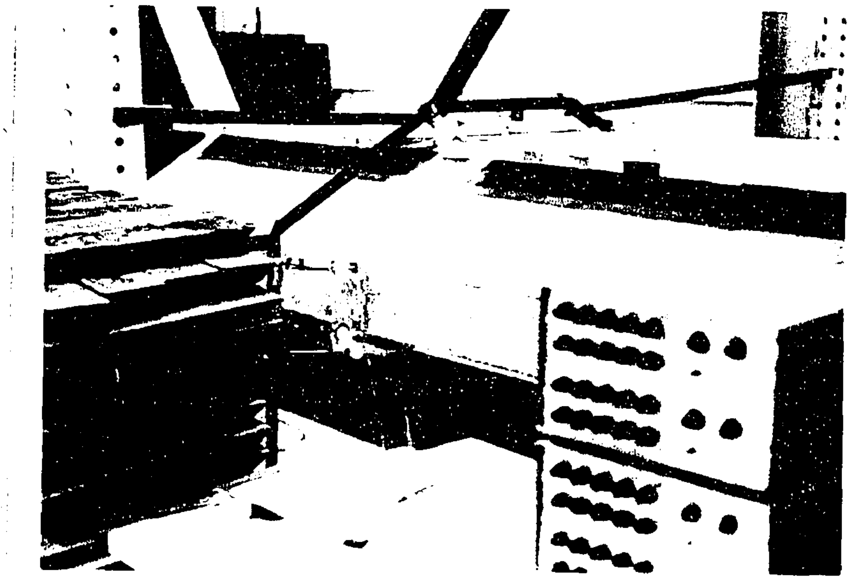


(a)

Fig. 41. Concrete cylinders are tested to determine the modulus of elasticity E_c. The resulting curve is given in Fig. 51. The types of steel (3/8"Ø, 3/16 Ø") which were used in the structure are also shown in these pictures.

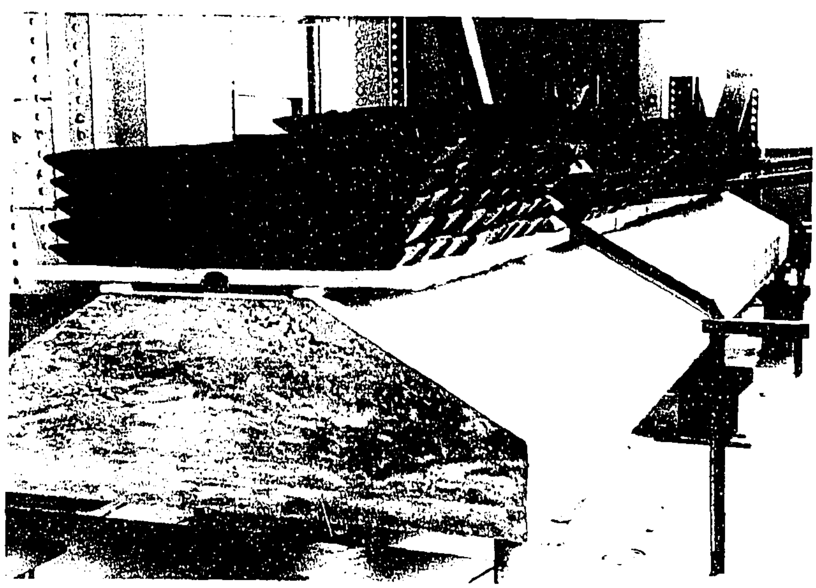


(b)

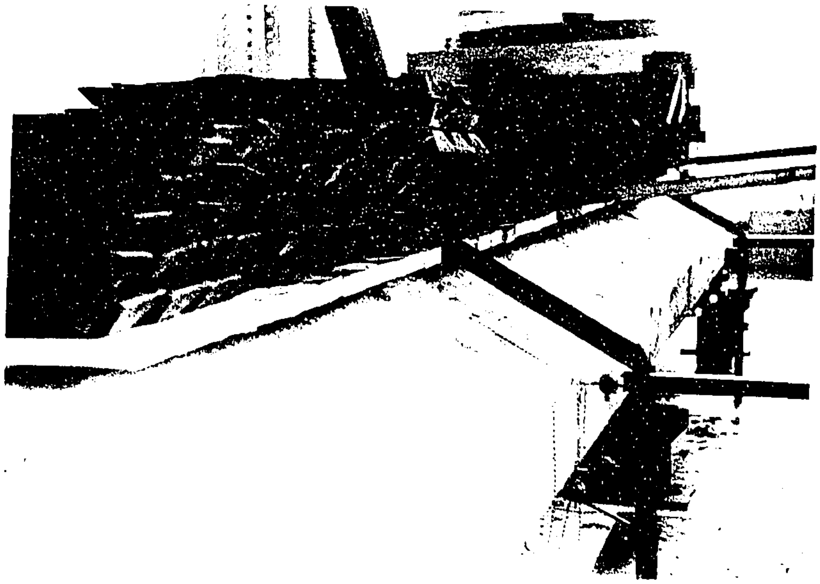


(a)

Fig. 42. Pieces of pig iron (each weighing about 40 lbs.) are loaded on a wooden platform. The uniform load is transferred to edges b and c of the structure through 1" x 2" wood strips.

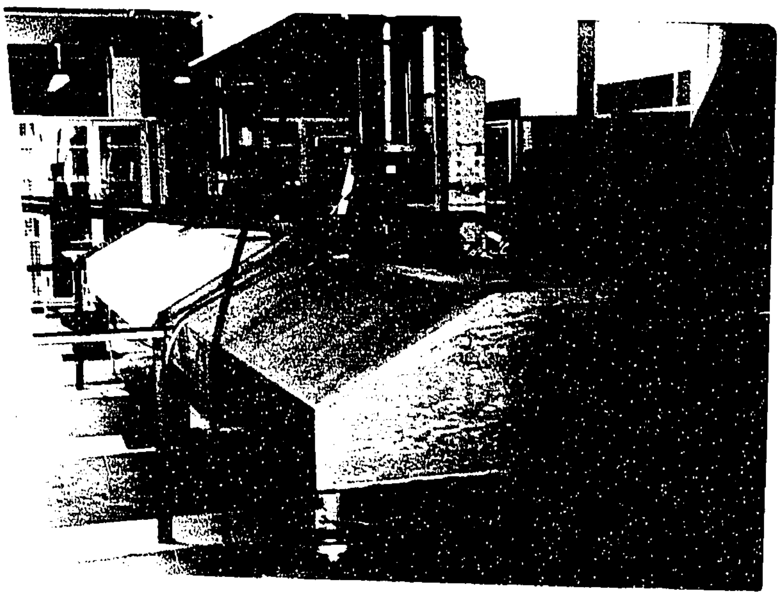


(b)



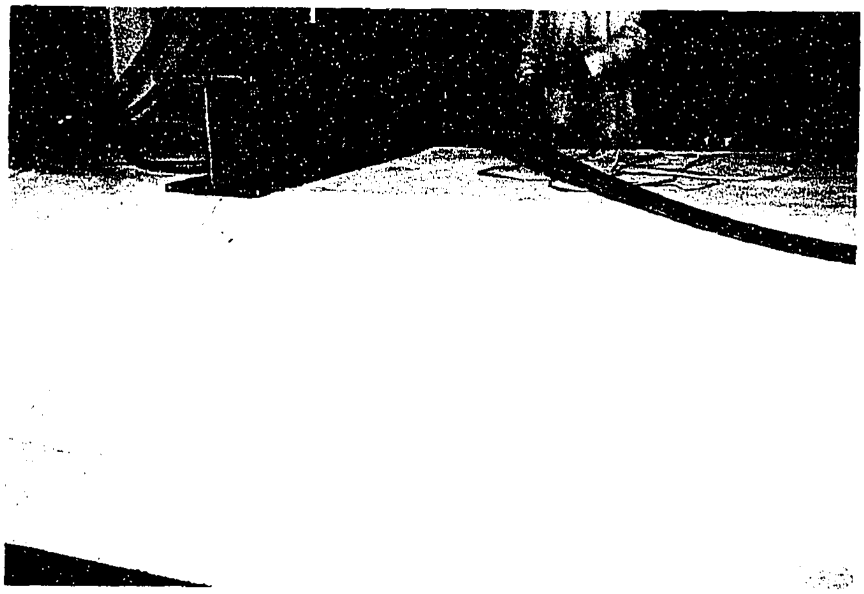
(a)

Fig. 43. The picture below shows pressure cylinders which may be used to apply concentrated loads, at edge b and c of the structure.



(b)

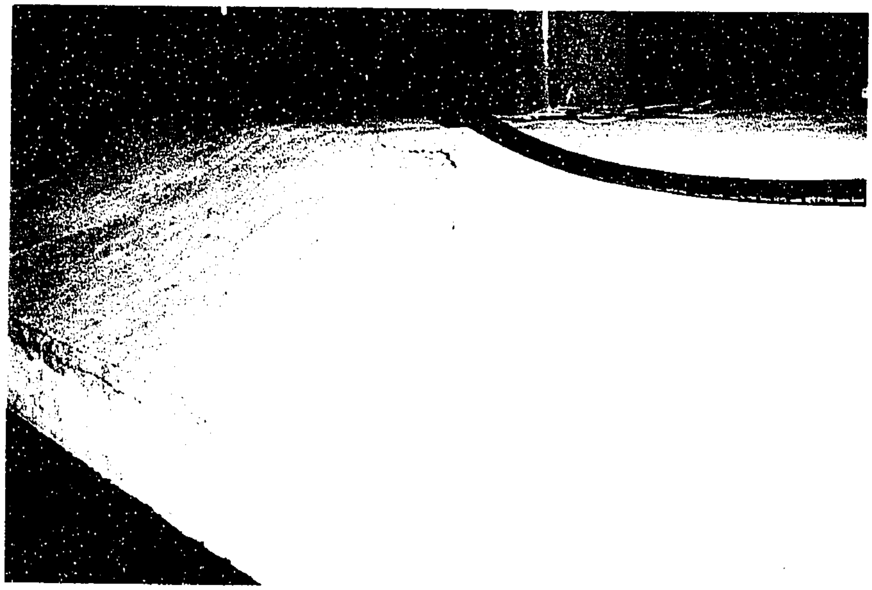
Fig. 44. A simply supported folded plate structure is loaded to failure.



(a)

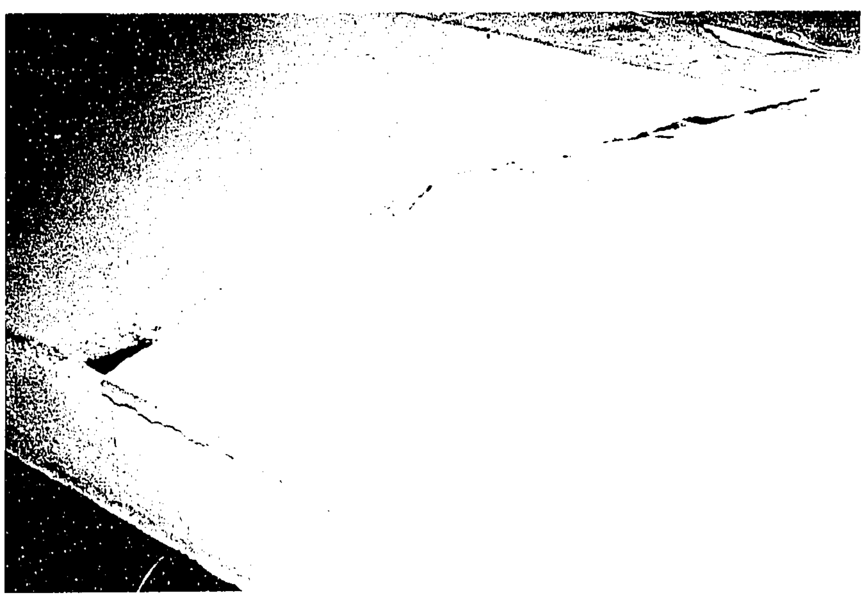
Fig. 46. Typical Pattern of cracks in folded plate structures.

(b)



(a)

Fig. 46. Typical cracks along line of longitudinal shear.



(b)

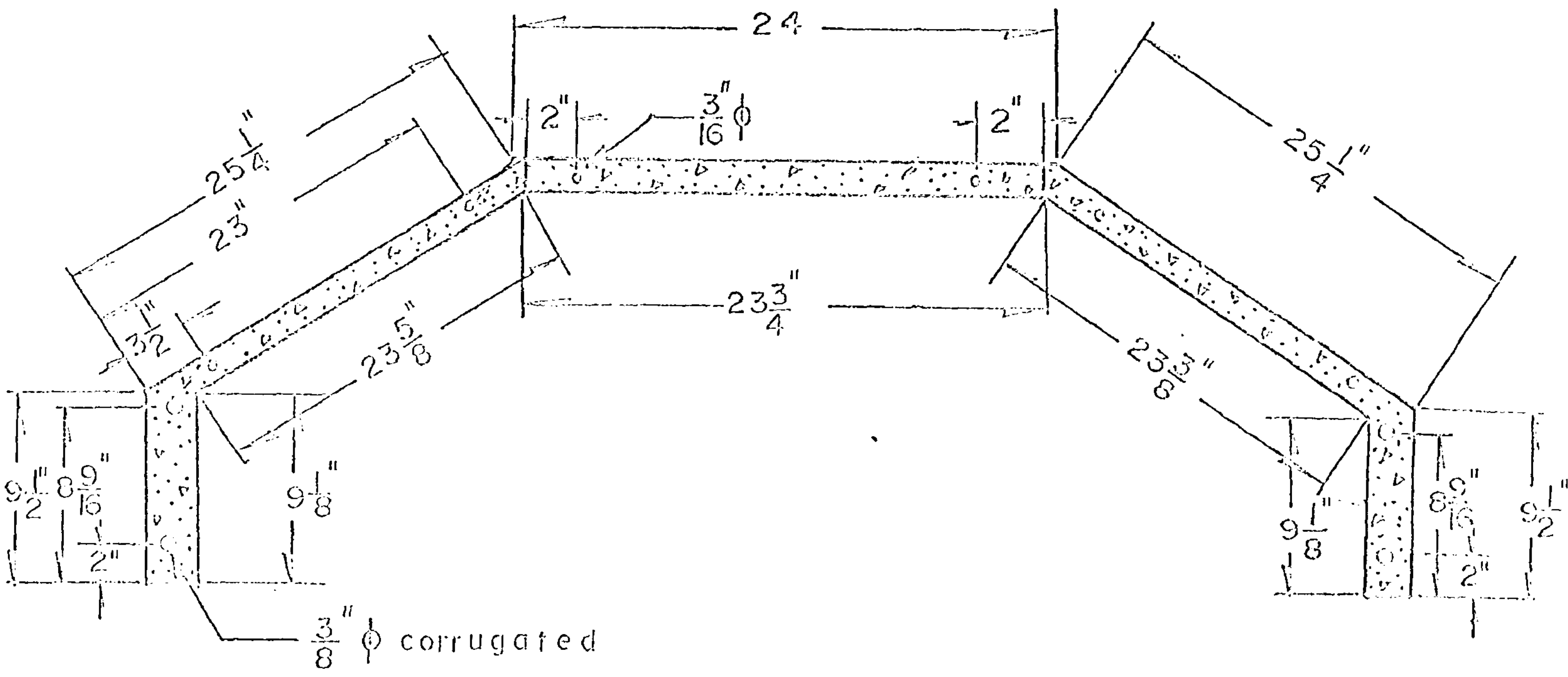


FIG. 47. CROSS-SECTION OF CONCRETE SPECIMEN

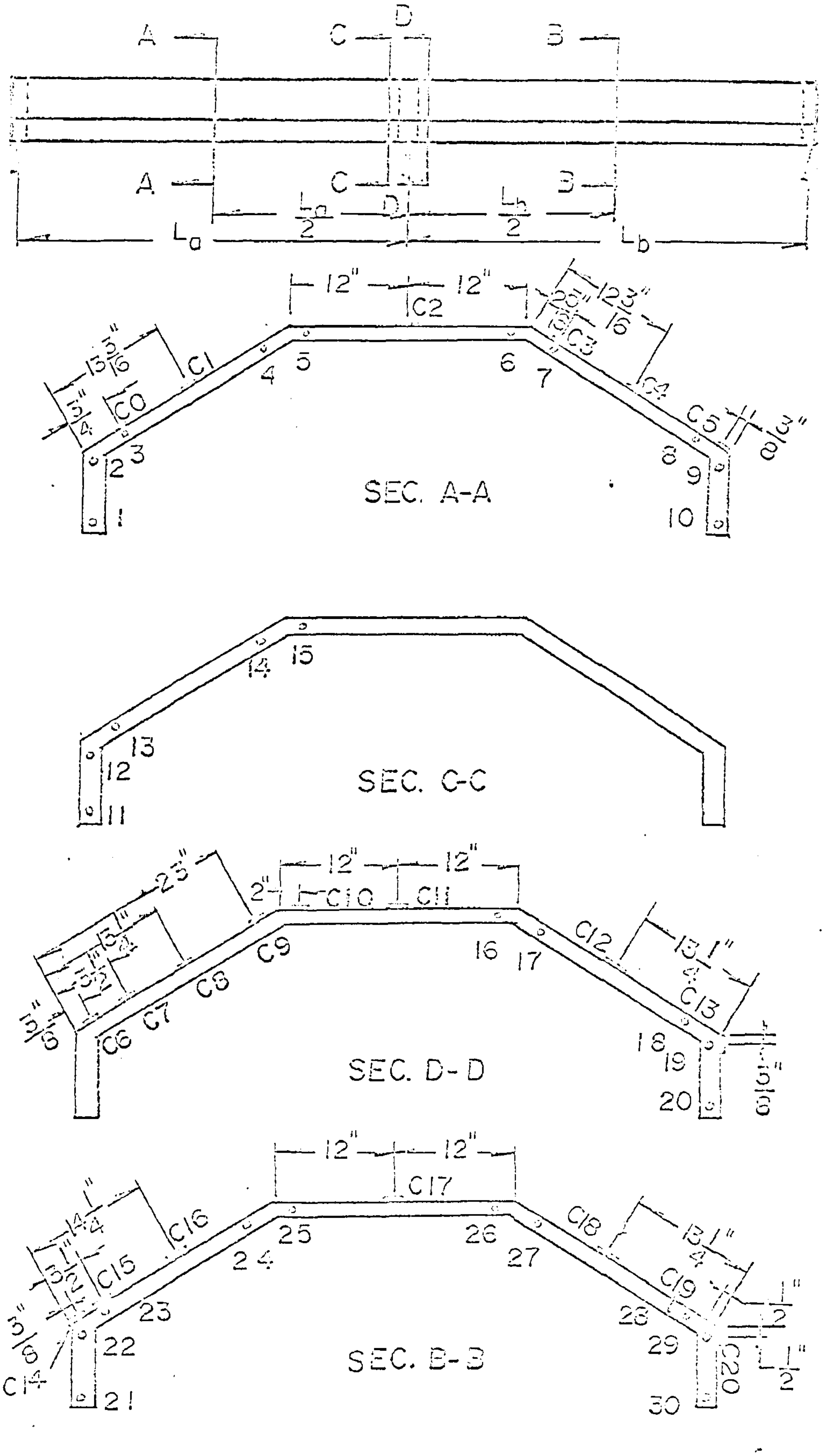


FIG. 46. LOCATION OF STRAIN GAUGES

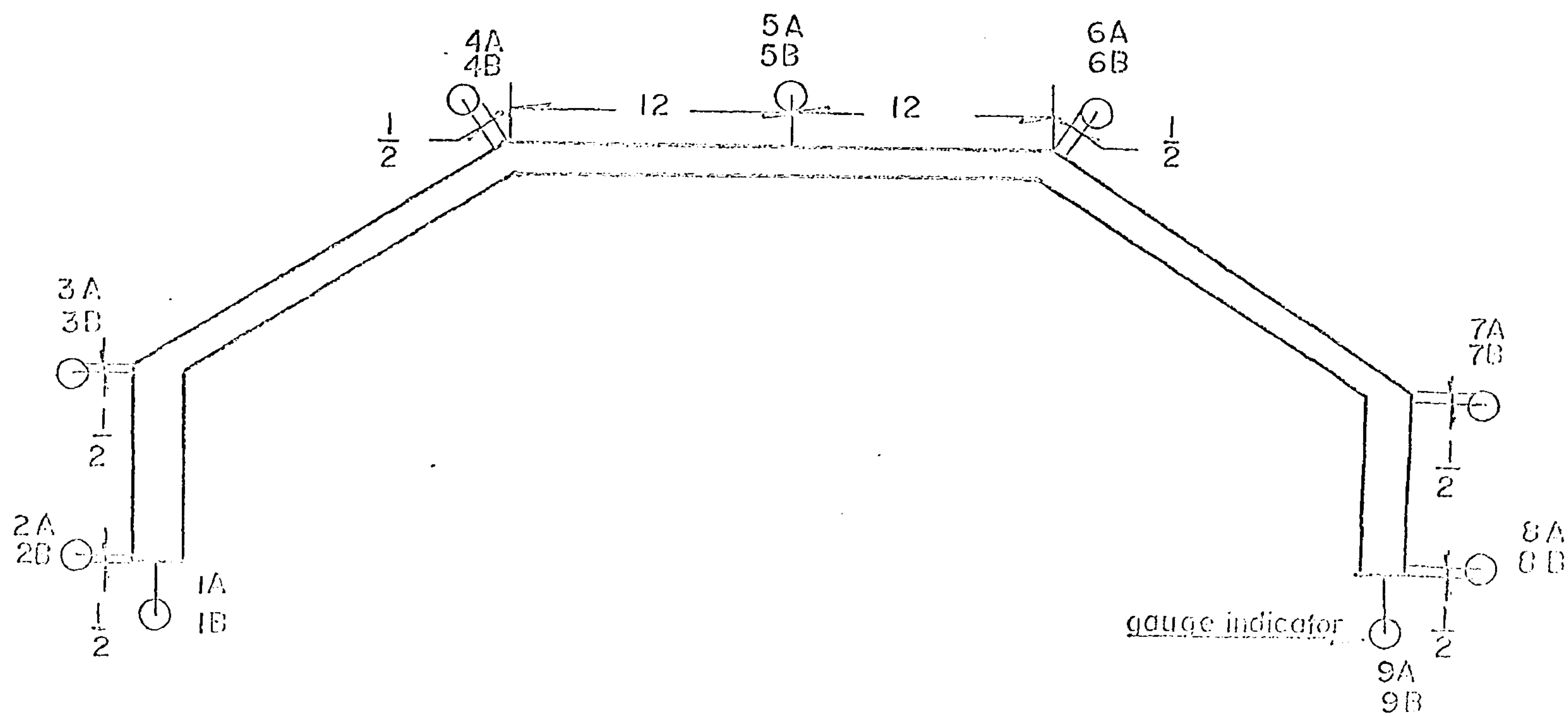


FIG. 49. SETUP OF GAUGE INDICATORS
AT MIDSPAN SECTIONS A & B
(TABLE II-2)

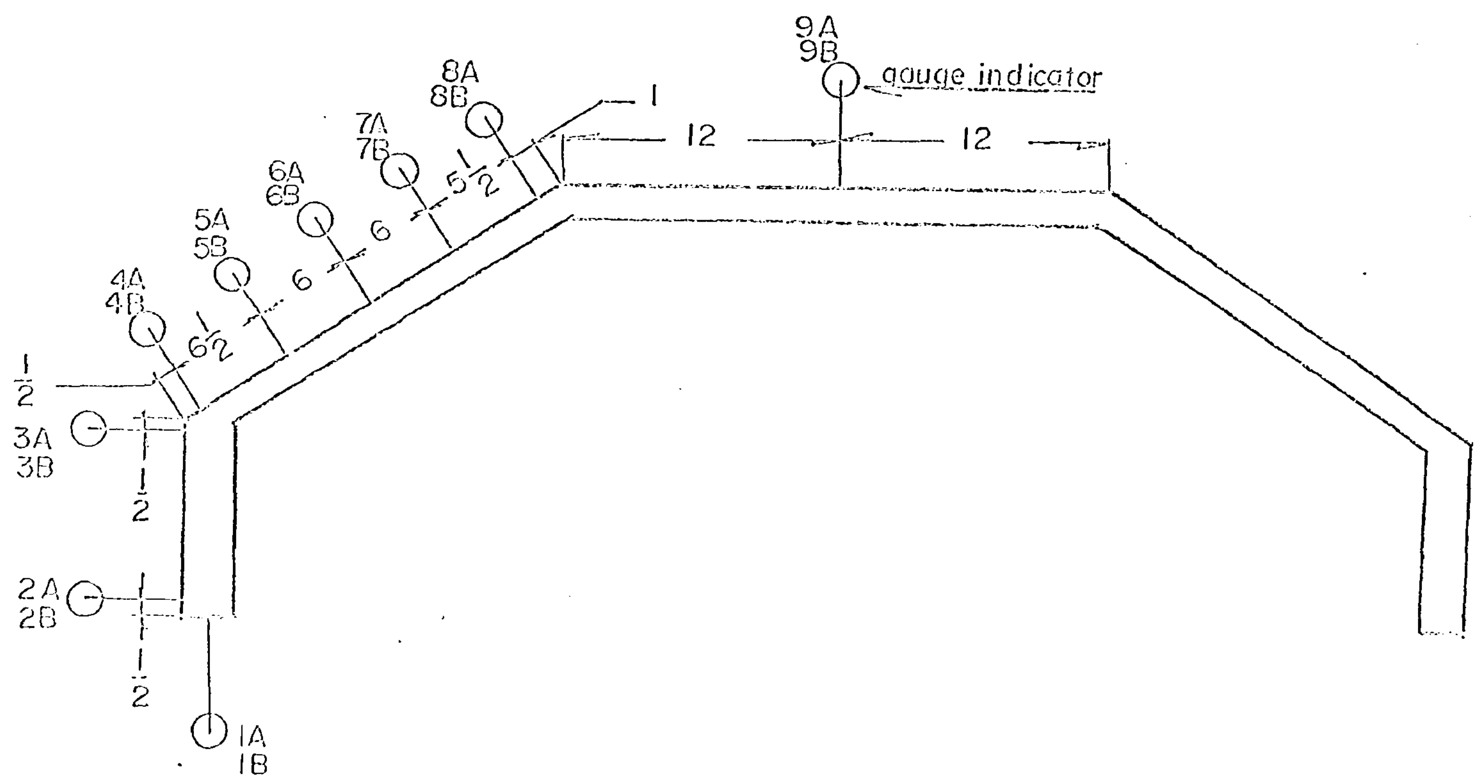


FIG. 50. SETUP OF GAUGE INDICATORS
AT MIDSPAN SECTIONS A & B
(TABLE 11-3)

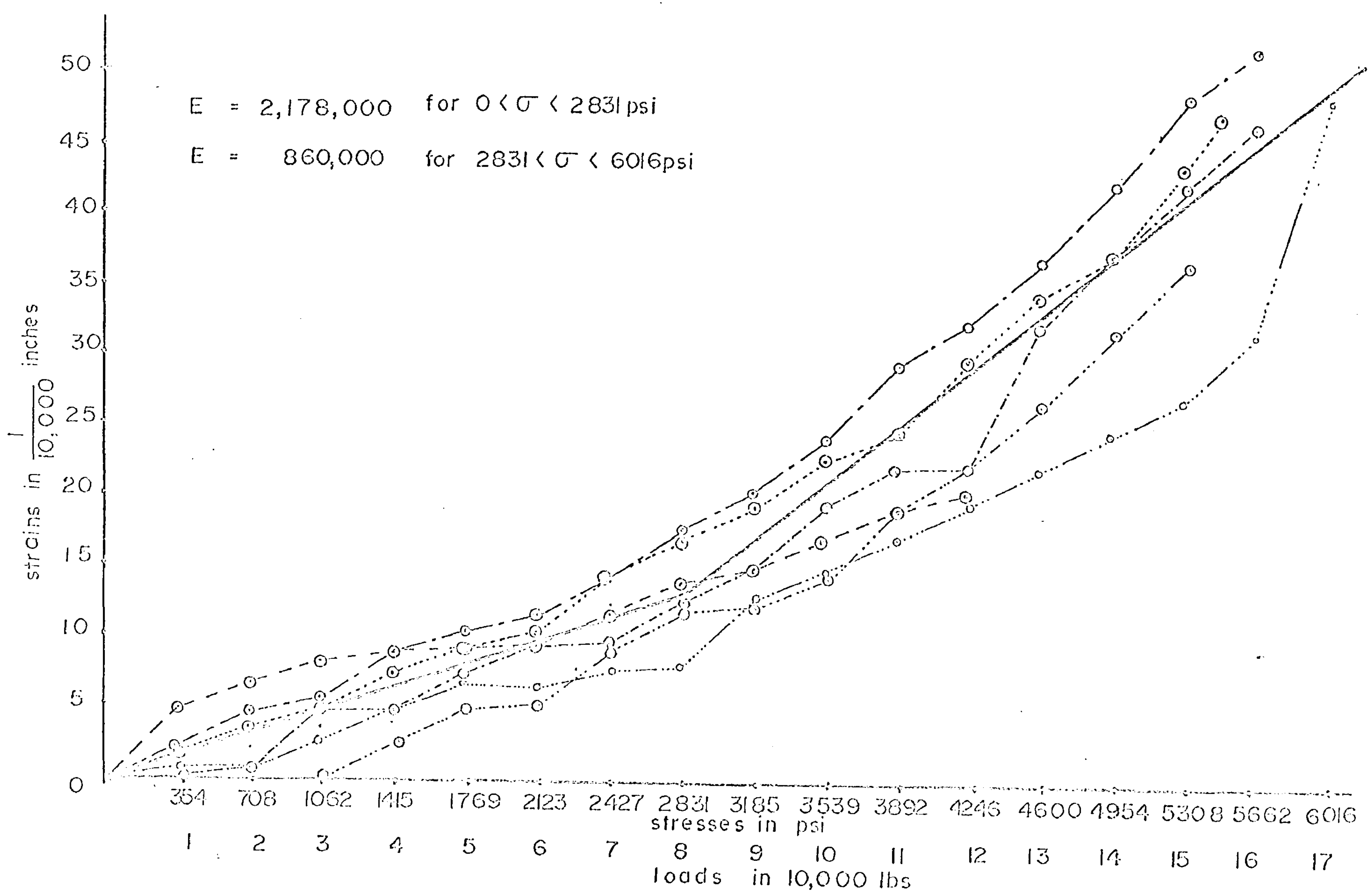


FIG. 51. MODULUS OF ELASTICITY OF CONCRETE

RESULTS OF STRAIN GAUGES (Strains in $\frac{\text{microinches}}{\text{Inch}}$)

GAUGE	167 lb/ft	334 lb/ft	500 lb/ft	668 lb/ft	835 lb/ft	1000 lb/ft
1	+4	+9	+14	+18	+30	+47
2	+13	+28	+42	+56	+70	+88
3	+14	+26	+36	+4[illegible]	+54	+66
4	-8	-18	-28	-36	-44	-54
5	-14	-26	-35	-43	-54	-65
6	-14	-28	-40	-52	-68	-81
7	-8	-18	-28	-36	-44	-54
8	+14	+25	+46	+48	+55	+66
9	+16	+29	+43	+56	+73	+90
10	+6	+10	+17	+20	+30	+47
11	+17	+34	+51	+69	+84	+121
12	-6	-19	-32	-48	-64	-90
13	+26	+53	+80	+93	+122	+170
14	+117	+222	+320	+425	+530	+616
15	+134	+259	+364	+468	+570	+630
16	+55	+109	+159	+209	+250	+314
17	+52	93	131	155	193	+270
18	-1	-1	+2	+5	+32	+49
19	-8	-20	-34	-48	-68	-82
20	+27	+62	+104	+152	+206	+284
21	+7	+14	+20	+25	+33	+37
22	+16	+30	+46	+59	+71	+89
23	+12	+22	+32	+44	+54	+65
24	-11	-23	-31	-40	-50	-63
25	-12	-21	-30	-38	-69	-85
26	-13	-24	-35	-45	-54	-68
27	-16	-20	-26	-35	-40	-55
28	+11	+20	+32	+42	+52	+66
29	+16	+24	+37	+64	+68	+77
30	+10	+20	+34	+45	+60	+75

TABLE 11-1

Strains in $\frac{\text{Microinches}}{\text{Inch}}$

(continued)

GAUGE	167 lb/ft	334 lb/ft	500 lb/ft	668 lb/ft	835 lb/ft	1000 lb/ft
c0	14	31	48	60	72	82
c1	2	4	6	11	12	15
c2	-13	-26	-39	-50	-63	-74
c3	-13	-27	-41	-53	-66	-77
c4	-2	-4	-5	-6	-4	-2
c5	17	32	46	60	73	87
c6	-34	-71	-109	-142	-179	-224
c7	-17	-36	-51	-69	-89	-110
c8	-4	-7	-11	-16	-22	-26
c9	12	21	32	42	50	63
c10	31	41	55	67	76	87
c11	17	34	51	66	82	106
c12	-3	-4	-5	-5	-4	-5
c13	-25	-55	-83	-107	-145	-177
c14	19	33	56	73	95	123
c15	12	22	32	43	52	68
c16	-1	-1	-2	-1	-3	0
c17	-12	-26	-37	-45	-59	-77
c18	-1	-1	-2	-2	-3	-3
c19	18	33	47	61	73	96
c20	21	42	61	81	100	120
A0	-18	-39	-65	-89	-114	-142
A1	-60	-120	-150	-192	-230	-269
A2	-32	-70	-108	-140	-172	-206
B0	-52	-104	-160	-196	-238	-290
B1	-52	-96	-136	-176	-213	-260
B2	-8	-15	-25	-51	-73	-95
1	100	220	350	465	590	
2	115	240	400	525	655	
3	118	250	400	540	685	
4	105	225	350	480	610	

TABLE 11-1

Strains in $\frac{\text{Microinches}}{\text{Inch}}$

o SR4 gauges applied on the reinforcing steel

○ SR4 gauges applied on the concrete surface

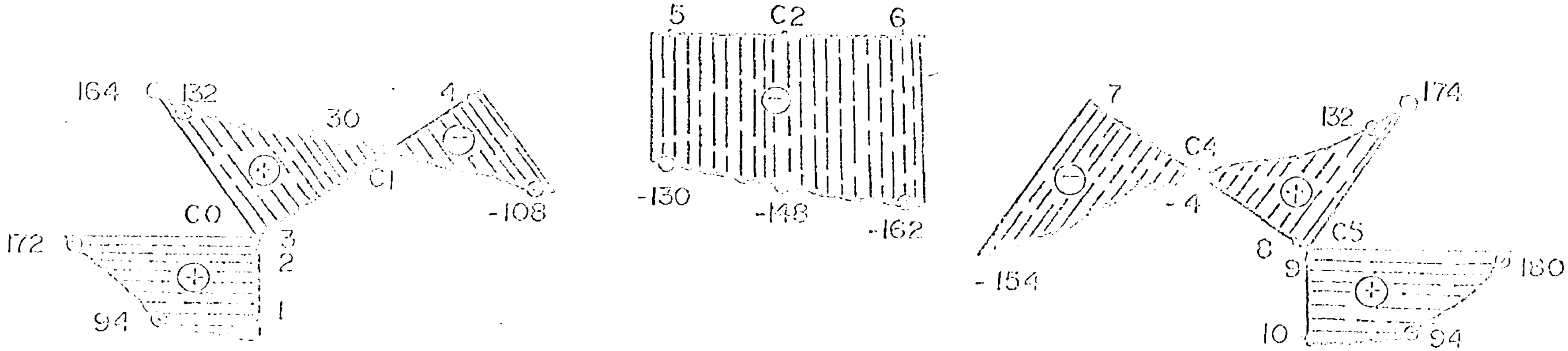


FIG. 52. STRESS DISTRIBUTION IN psi AT SECTION A-A (FIG.48)
DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED
AT EDGES 2 & 3 OF BOTH SPANS

O SR 4 gauges applied on the reinforcing steel

O SR 4 gauges applied on the concrete surface

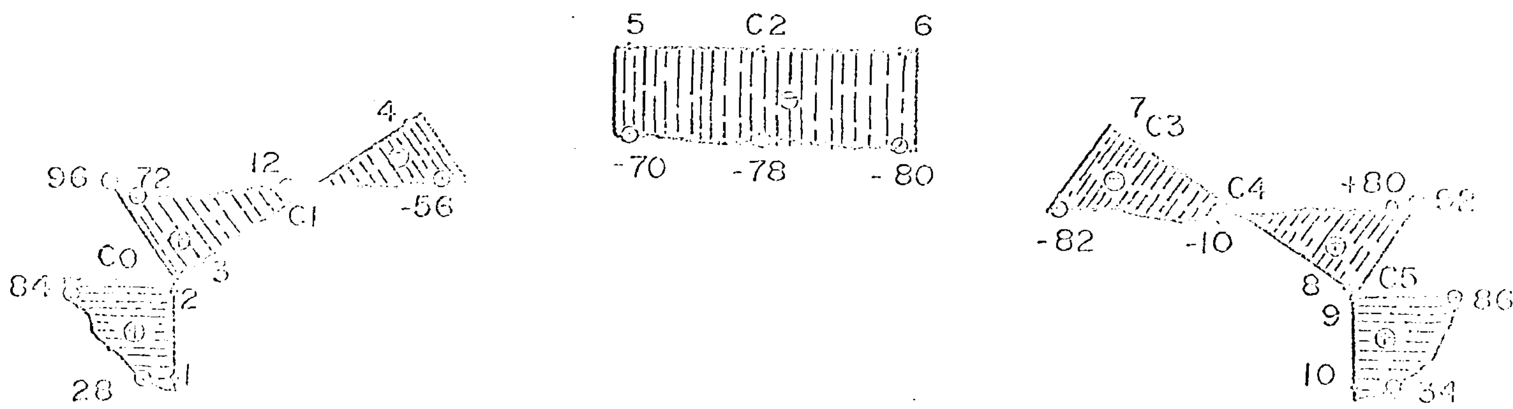


FIG. 53. STRESS DISTRIBUTION IN psi AT SECTION A-A (FIG. 48) DUE TO UNIFORM LOADS OF 500 lbs/ft APPLIED AT EDGES 2 & 3 OF BOTH SPANS

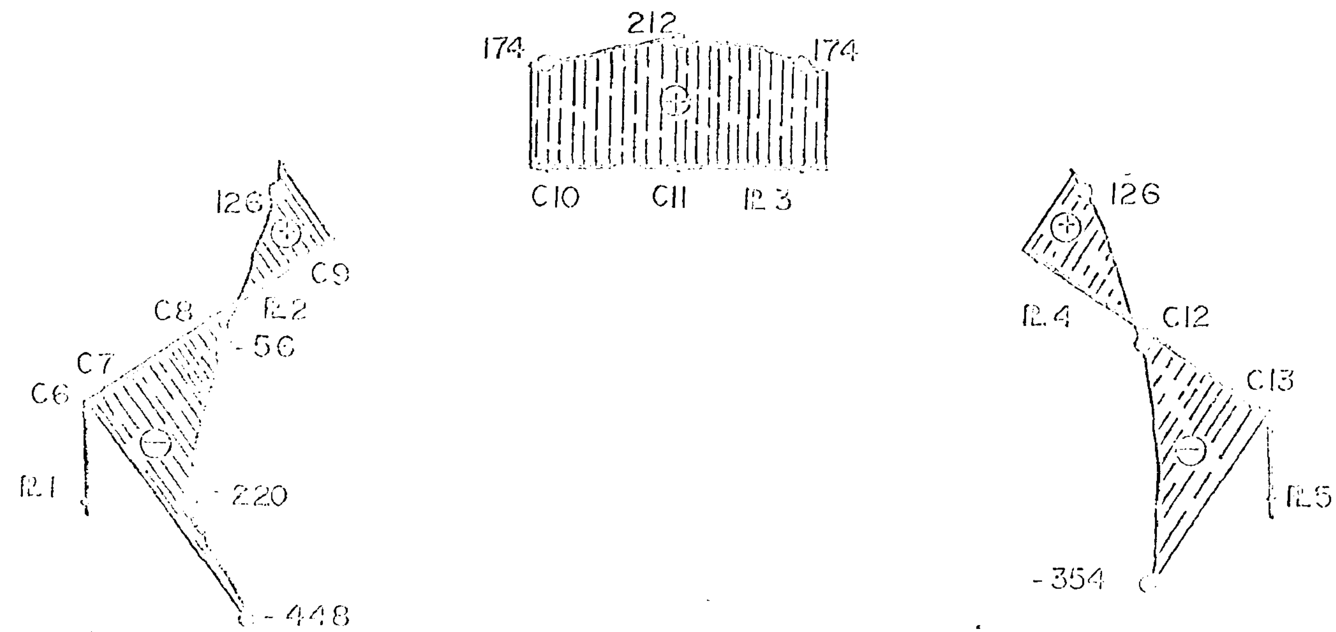


FIG. 54. STRESS DISTRIBUTION IN psi AT THE MIDDLE SUPPORT SECTION DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED AT EDGES 2 & 3 OF BOTH SPANS. THE STRESS READINGS ABOVE ARE FROM SR4 GAUGES APPLIED ON THE CONCRETE SURFACE.

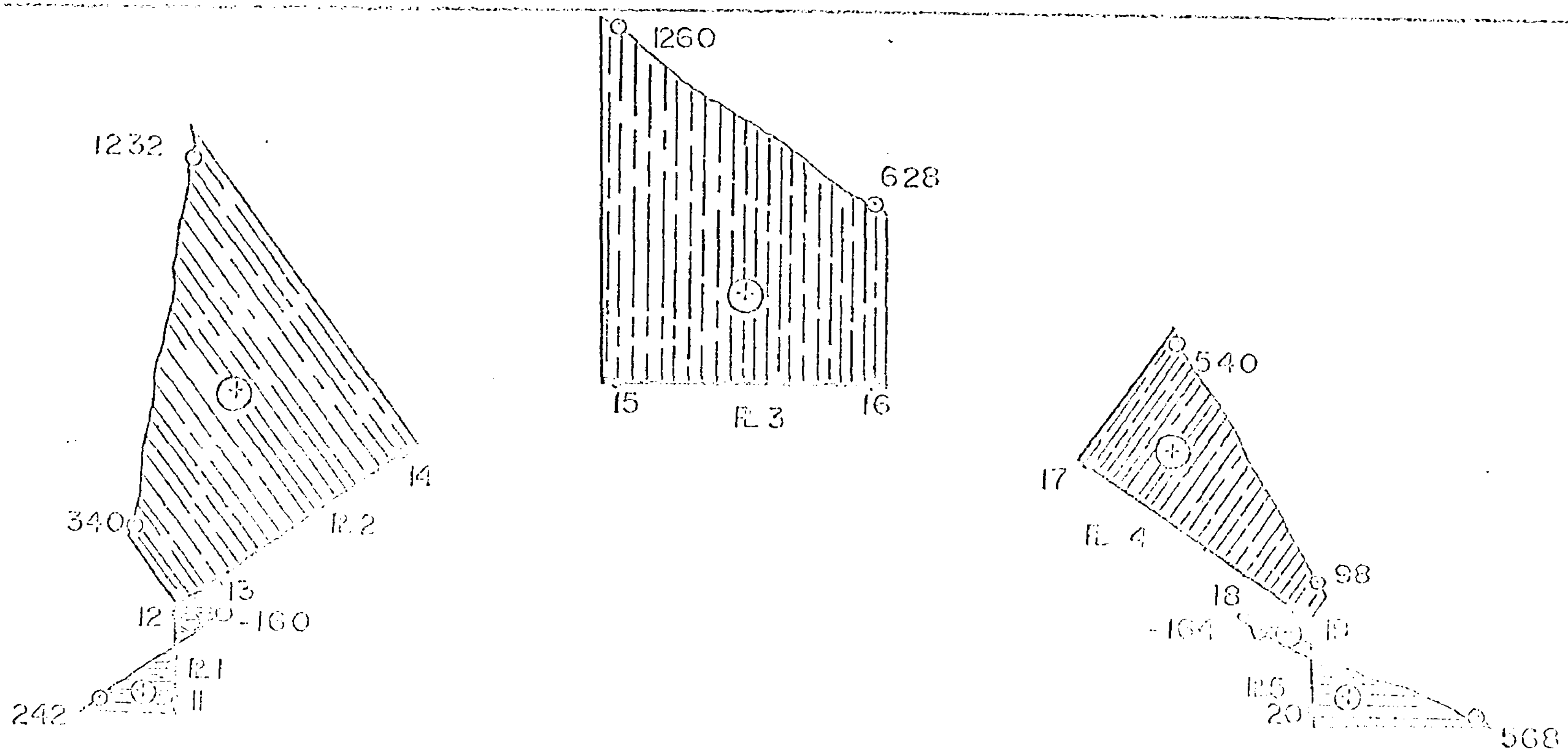


FIG. 55. STRESS DISTRIBUTION (psi) IN THE REINFORCING STEEL AT SECTION C-C AND D-D (FIG. 48) DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED AT EDGES 2 & 3 OF BOTH SPANS

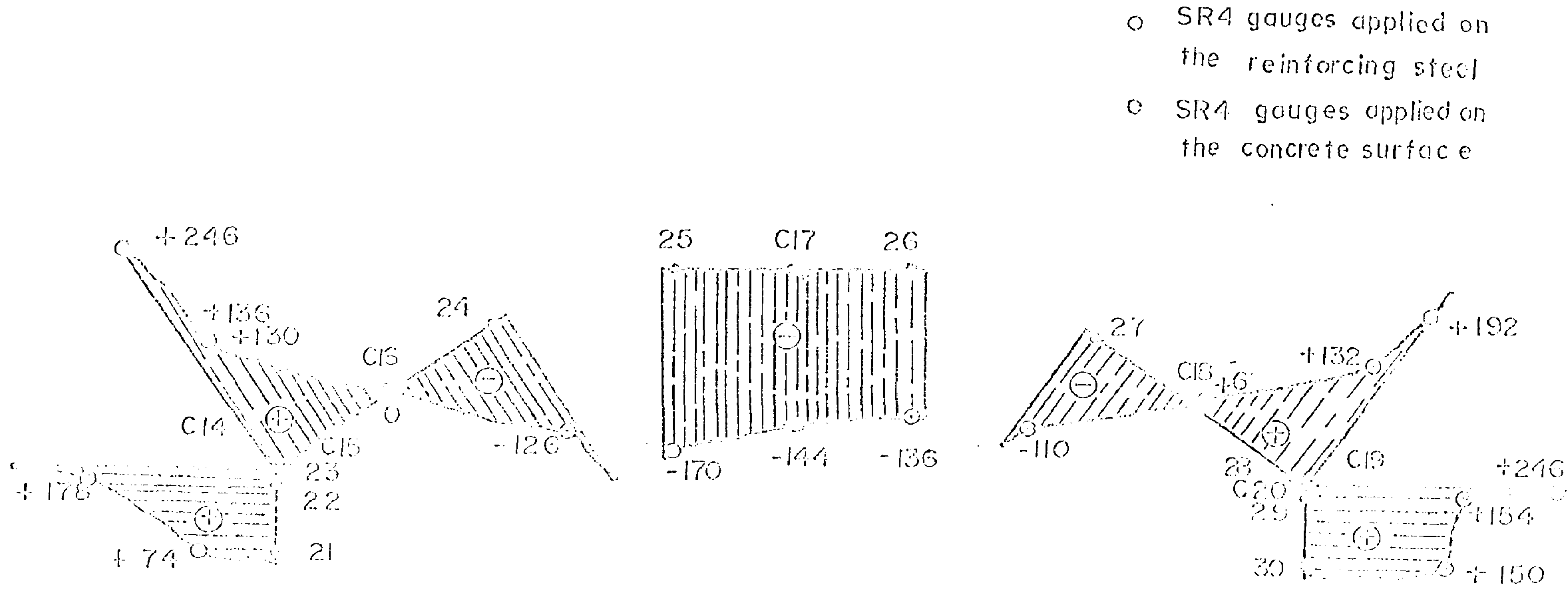


FIG. 56. STRESS DISTRIBUTION (psi) AT SECTION B-B (FIG. 48) DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED AT EDGES 2&3 OF BOTH SPANS

READING OF DIAL INDICATORS (Deflections in $\frac{1}{1000}$ in)

G. No. \ lb/ft	167	334	500	668	835	1000
1A	.5	-.5	-1.2	-3.0	-4.5	-6.0
2A	8.2	16.8	24.7	32.0	40.0	48.5
3A	4.0	9.5	14.5	20.8	26.3	31.0
4A	-5.0	-10.8	-16.0	-21.0	-25.0	-30.0
5A	-11.0	-20.5	-29.0	-38.0	-46.5	-57.0
6A	-11.0	-19.5	-27.0	-34.0	-42.5	-52.0
7A	7.5	13.1	10.0	24.5	30.0	37.0
8A	9.8	19.9	28.5	37.8	52.0	60.0
9A	.3	-1.9	-4.0	-6.1	-9.0	-11.0
1B	.5	-.1	-1.0	-2.0	-2.5	-3.5
2B	8.0	16.1	23.5	30.7	37.5	46.5
3B	6.0	11.2	16.0	20.4	24.8	30.5
4B	-4.5	-10.1	-15.0	-19.9	-25.0	-30.0
5B	-9.0	-18.5	-27.0	-35.1	-42.8	-43.5
6B	-4.5	-11.0	-18.0	-24.9	-30.0	-40.0
7B	6.0	9.7	13.5	17.5	20.5	25.0
8B	8.0	14.9	21.9	28.5	34.0	48.0
9B	0	-.5	-1.0	-1.5	-2.0	-3.0

TABLE 11-2

READING OF DIAL INDICATORS (Deflection in $\frac{1}{1000}$ inches)

lb/ft / G. No.	167	334	500	668	835
1A	-1.6	-3.0	-5.0	-6.8	-8.5
2A	8.0	16.5	23.2	29.5	35
3A	6.0	13.0	17.0	21.5	25
4A	2.5	6.0	8.5	11.5	13.5
5A	1.8	4.0	6.8	8.5	10.0
6A	-.2	0	.3	1.7	1.3
7A	-1.5	-2.8	-3.7	-5.7	-6.2
8A	-5.0	-8.0	-11.0	-15.0	-19.0
9A	-10.5	-21.0	-30.0	-37.5	-44.0
1B	.3	-.5	-1.2	-2.3	-3.5
2B	9.0	17.2	24.0	30.0	36.0
3B	6.3	12.8	17.8	21.3	24.8
4B	2.5	6.0	8.5	11.5	13.5
5B	.5	2.5	2.7	4.2	6.5
6B	-1.0	-2.0	-3.0	-3.5	-3.0
7B	-4.0	-7.0	-10.0	-12.0	-14.0
8B	-6.0	-10.5	-15.0	-19.8	-22.0
9B	-12.0	-22.5	-30.8	-38.0	-45.5

TABLE 11-3

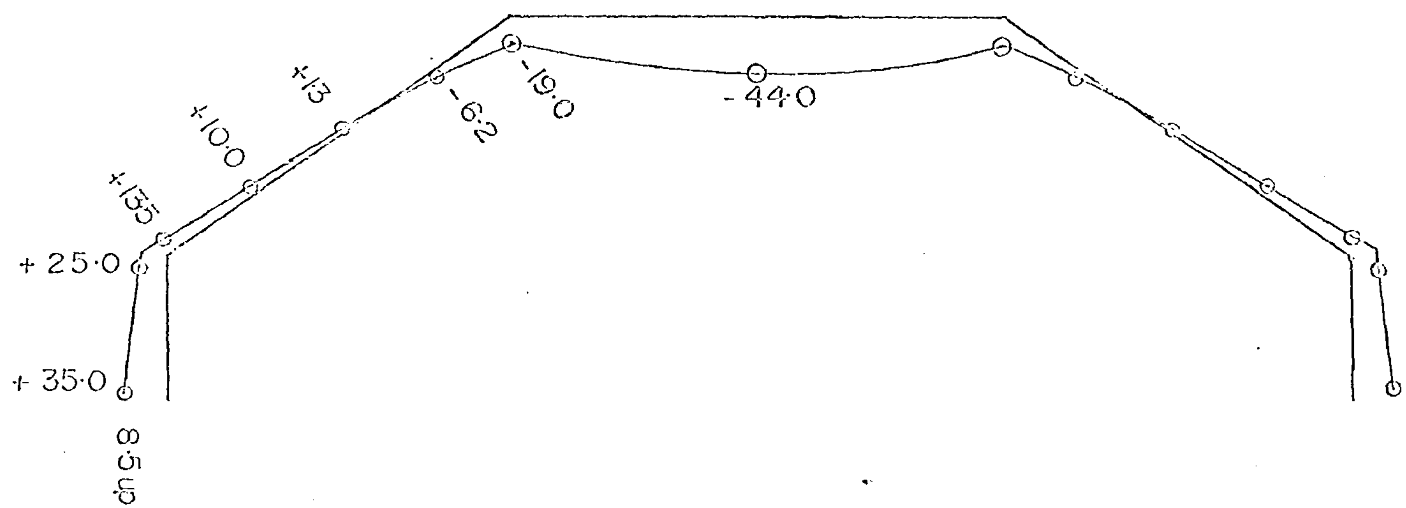


FIG. 57. EXPERIMENTAL DEFLECTIONS AT SECTION A-A (FIG. 48) DUE TO UNIFORM LOADS OF 1000 lbs/ft APPLIED AT EDGES 2 & 3 OF BOTH SPANS. THE READINGS ARE IN $\frac{1}{10000}$" (table II-3)

CHAPTER XII

CONCLUSION

The experimental program described in Chapter XI yields relatively consistant results. The concrete prismatic structure was subjected to repeated identical tests in order to obtain a realistic pattern of deflection and stress distribution. The results are shown in Tables 1-1, 1-2, 1-3, and Figs. 52 to 57.

No comparison can be made between the experimental results and analytical results obtained from equations developed from Flügge's theory. In the deflection equation (8-7) we have the factor $(S_m - h_m \langle T'_{b(m-1)} + T'_{bm} \rangle)$ where S_m is the uniform load and $T'_{b(m-1)}$ as well as T'_{bm} are constants related to the longitudinal shears. When equation (8-7) is solved the uniform load S_m produces certain deflection in the direction in which it itself acts (Fig. 58) while the shear constants tend to produce a deflection opposite to that of S_m. The sum of the two deflections should be smaller than and in the same direction as the one produced by S_m. These shearing constants from Flügge's theory, however, seem to be too large. As a result the term $h_m \langle T'_{b(m-1)} + T'_{bm} \rangle$ is larger than S_m thus upsetting the direction and numerical value of the deflection.

If the shearing constants could logically be made smaller the whole theory would seem to conform with experimental results. It is therefore necessary to carry on further studies on the longitudinal shearing stresses in

Flügge's theory and then apply the results to the theory for continuous folded plates developed in this paper. After the revision has been made the same theory may be used for any kind of loading condition by expressing these loads in Fourier series.

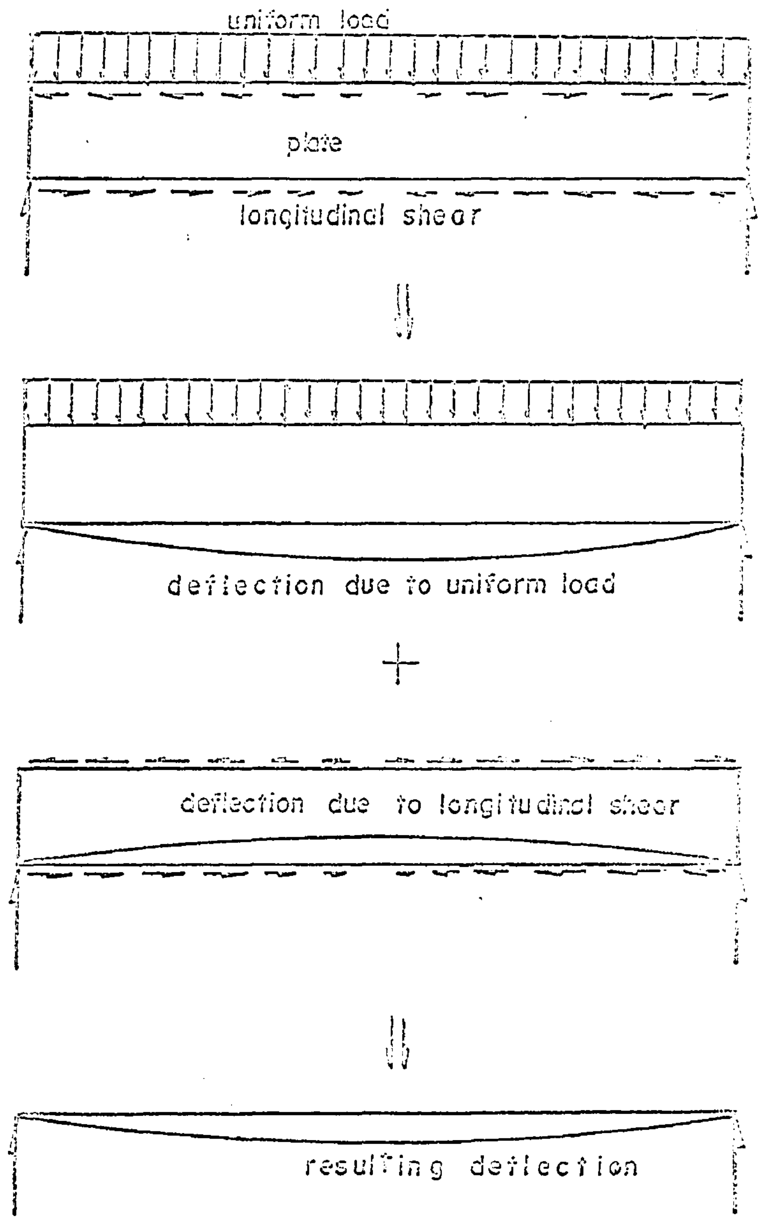


FIG. 58. RELATION OF DEFLECTIONS DUE TO S_m AND T_m

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1959 In September, 1959, he enrolled at Assumption University of Windsor, Windsor, Ontario, Canada, to study Civil Engineering.

1963 In May, 1963, he was graduated from Assumption University of Windsor with a Bachelor of Applied Science degree. In September, 1963, he continued his studies at the University of Windsor in order to obtain the degree of Master of Applied Science in Civil Engineering.